Chapter 10. New Combinatorial Techniques

Note. Given an oriented knot/link diagram, we modify an individual crossing in a particular way. If the crossing is left-handed then we can change it to right-handed. We denote this modification of a crossing as L_-. If the crossing is right-handed then we can change it to left-handed. We denote this modification of a crossing as L_+. A crossing can be *smoothed* in which case small sections of arcs are removed at the crossing and the resulting segments are joined in a way that removes the crossing and preserves the orientation. There is only one way to perform the smoothing operation, so it is well-defined. We denote this modification of a crossing as L_S (another common notation is L_0). Figure 10.1 gives three diagrams where the marked crossing is right handed (left, we denote this as L_+), the crossing is left handed (center, we denote this as L_-), and the crossing has been smoothed (right, we denote this as L_S).

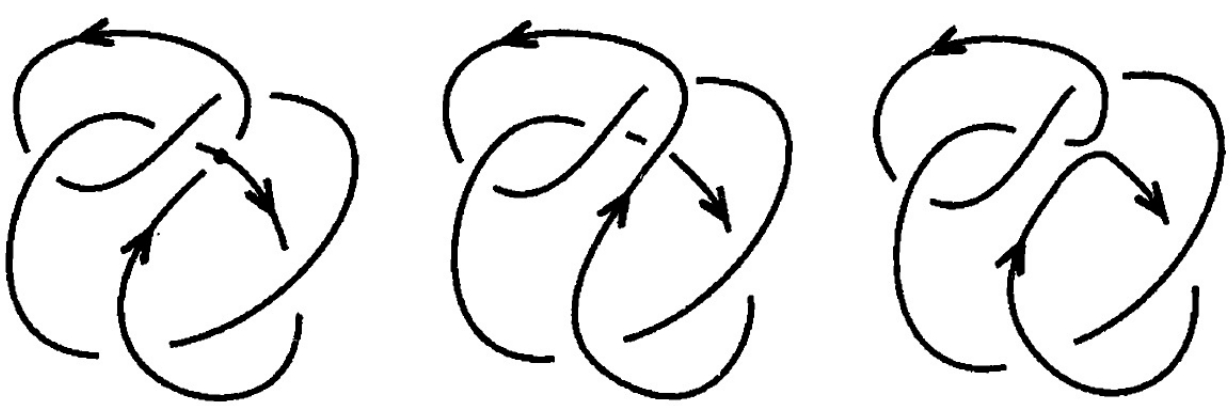

Figure 10.1

Note 10.0.A. James W. Alexander himself proved that the Alexander polynomials (if computed "appropriately," as Livingston says on page 205) are related by the equation $A_{L_+}(t) - A_{L_-}(t) = (1 - t)A_{L_S}(t)$. Notice that the ambiguity of a factor of $\pm t^k$ (where $k \in \mathbb{Z}$) as given in Theorem 3.5.6 does not necessarily allow us to compute one of the Alexander polynomials $A_{L_+}(t)$, $A_{L_-}(t)$, and $A_{L_S}(t)$ from the other two. What we need is a way to "normalize" the Alexander polynomial, which we will soon have (this is the computed "appropriately" comment of Livingston).

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