Section 2.2. The Definition of a Knot

Note. In this section we consider representing knots as polygonal curves in 3-space.

Definition. For any distinct points in 3-space (i.e., in $\mathbb{R}^3$), p and q, let $[p, q]$ denote the line segment joining p and q. For an ordered n-tuple of distinct points $(p_1, p_2, \ldots, p_n)$, the union of the segments $[p_1, p_2], [p_2, p_3], \ldots, [p_{n-1}, p_n], [p_n, p_1]$ is a *closed polygonal curve* in 3-space. If each segment intersects exactly to other segments (intersecting each only at an endpoint), then the curve is *simple*.

Definition. A *knot* is a simple closed polygonal curve in 3-space.

Note. Figure 2.4 gives a polygonal curve representing the trefoil knot (in Figure 2.4(a)) and the unknot (in Figure 2.4(a)).

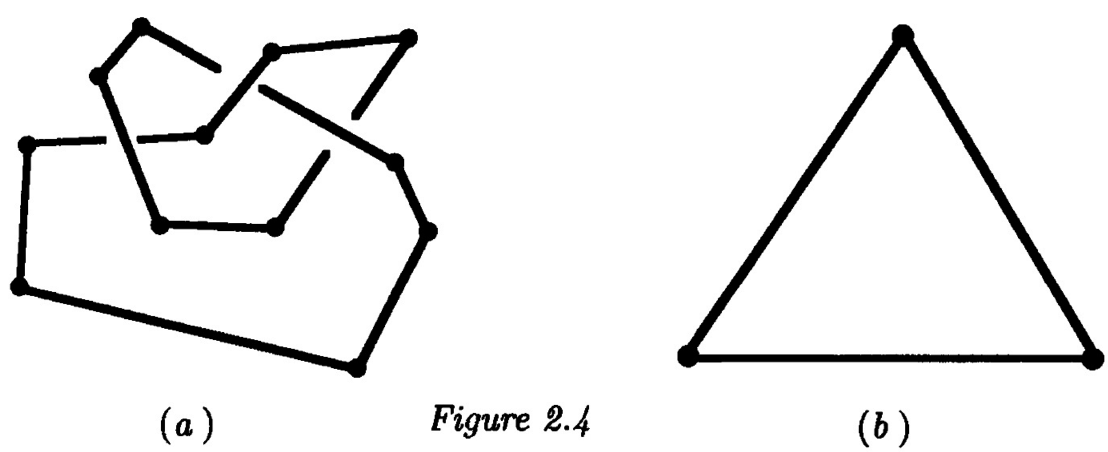

Figure 2.4

The knots we draw in this course will be drawn "smooth," but each can be easily represented as a polygonal curve (and vice-versa).

Note. Of course a knot is not defined by a unique n-tuple of point. For one thing, we can cycle the n-tuple around and get exactly the same polygonal curve. We can also eliminate any points that produce consecutive collinear segments (or add points that result in subdividing a segment). When the number of points is a minimal collection of points determining a polygonal curve representing a knot, we have the following definition.

Definition. If the ordered n-tuple $(p_1, p_2, \ldots, p_n)$ of points in 3-space defines a knot, and no proper ordered subset defines the same knot, the elements of the set $\{p_i\}$ are the *vertices* of the knot.

Definition. A *link* is the finite union of disjoint knots. The *unlink* is a union of disjoint unknots all lying in a plane.

Note. In the next section, we define the equivalence of knots and clear up some of the ambiguities with which we are currently working.

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