Section 2.3. Equivalence of Knots, Deformations

Note. In this section we define an elementary deformation (or a polygonal knot) and the idea of equivalence under such deformations.

Definition. A knot J is an *elementary deformation* of the knot K if one of the two knots is determined by a sequence of points $(p_1, p_2, \ldots, p_n)$ and the other is determined by the sequence $(p_0, p_1, p_2, \ldots, p_n)$, where (1) p_0 is a point which is not collinear with p_1 and p_n, and (2) the triangle spanned by (p_0, p_1, p_n) intersects the knot determined by $(p_1, p_2, \ldots, p_n)$ only in the segment $[p_1, p_n]$. (Here, by the "triangle spanned by (p_0, p_1, p_2)" we mean the the three edges and the interior of the triangular region with vertices p_0, p_1, and p_2.)

Note. Condition (1) in the above definition simply deals with noncollinearity of consecutive points. Condition (2) eliminates the possibility that while deforming the knot does not intersect itself. In Figure 2.5(a), an elementary deformation is given. In Figure 2.5(b), we see a manipulation that violates condition (2); notice that the line segment from lower left to upper right undercrosses the segment from upper left to lower right but that in the deformation this has changed to an overcrossing (so that the knots are different). Also, the lower left to upper right segment intersects the triangular surface, which is prohibited.

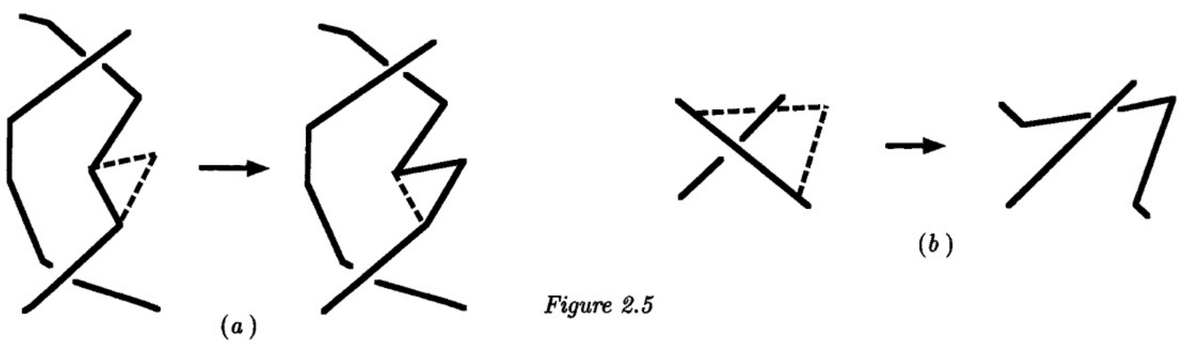

Figure 2.5

Note. We are now ready to use elementary deformations to define equivalent knots.

Definition. Knots K and J are *equivalent* if there is a sequence of knots $K = K_0, K_1, K_2, \ldots, K_n = J$, with each K_{i+1} an elementary deformation of K_i, for i greater than 0.

Note. It is easily verified that knot equivalence actually is an equivalence relation (i.e., reflexive, symmetric, and transitive). The study of knot theory is ultimately the study of the equivalence classes of knots under this equivalence relation. We usually do not speak of "the equivalence class of knots containing knot K" and instead merely speak of "knot K" when speaking of any element of the equivalence class.

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