Section 3.2. Colorings

Note. In this section we introduce the coloring of a knot, which involves associating colors with the arcs of (a diagram of) a knot. The condition of colorability can sometimes be used to show that two knots are not equivalent.

Definition. A knot diagram is *colorable* if each arc can be drawn using one of three colors (say red, denoted R, yellow, denoted Y, and blue, denoted B) in such a way that (1) at least two of the colors are used, and (2) at any crossing at which at least two colors appear, all three colors appear.

Note. Figure 3.4 illustrates a coloring of a knot diagram. Notice that there are 8 crossings. At 2 crossings only the color red appears and at 6 crossings all 3 colors appear.

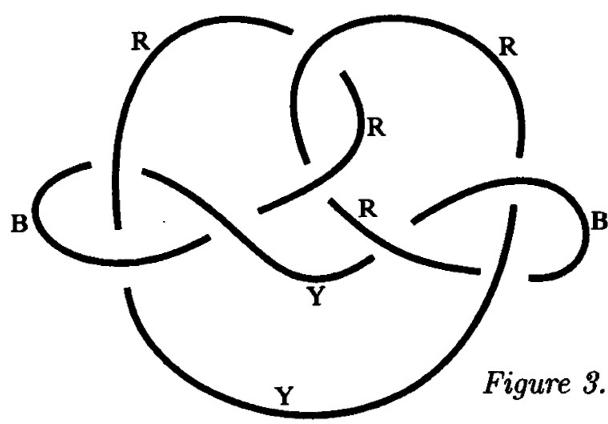

Figure 3.4

Note. We need a preliminary result before we extend the idea of colorability from diagrams of knots to knots themselves.

Theorem 3.2.2. If a diagram of a knot K is colorable, then every diagram of K is colorable.

Definition. A knot is called *colorable* if its diagrams are colorable.

Note. We can conclude from Theorem 3.2.2 that nontrivial knots exist. The trivial knot is not colorable because part (1) of the definition, which requires that at least colors are used, cannot be satisfied since the trivial knot only has one arc. So by Theorem 3.2.2, any colorable knot (such as the one given in Figure 3.4) is nontrivial. So colorability gives us one tool with which to potentially distinguish between knots.

Note. We now present a proof of Theorem 3.2.2.

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