Section 3.3. A Generalization of Colorability, mod p Labelings

Note. In this section we give a generalization of a the idea of a knot diagram coloring. This generalization is also unchanged by Reidemeister moves (see Theorem 3.3.3).

Definition. A knot diagram can be *labeled mod p* if each edge can be labeled with an integer from 0 to $p-1$ such that (1) at each crossing the relation $2x - y - z \equiv 0 \pmod{p}$ holds, where x is the label on the overcrossing and y and z the other two labels, and (2) at least two labels are distinct.

Note. Notice that if we take $p = 3$ and $\{x, y, z\} = \{0, 1, 2\}$, then we have that a labeling mod 3 is the same as a coloring of a knot diagram as defined in the previous section. It is straightforward to go through the various cases that can occur at a crossing. So a labeling mod p is a generalization of a coloring.

Note. Figure 3.11 illustrates a mod 7 labeling.

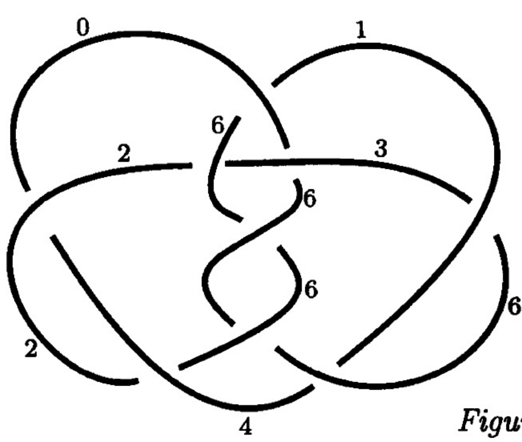

Figure 3.11

Note. The following result is to be proved in Exercise 3.3.3 by showing that if a knot diagram can be labeled mod p, then modifying the diagram by a Reidemeister move produces a knot diagram that also can be labeled mod p.

Theorem 3.3.3. Labeling Theorem.

If some diagram for a knot can be labeled mod p then every diagram for that knot can be labeled mod p.

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