## Chapter 4. Geometric Techniques

**Note.** The surface given in Figure 4.1 has a boundary that is a knot (in fact, it is a trefoil knot). We also present here a photo from Dave Richeson's Twitter page of a model of the same surface.



**Note.** In this chapter we consider properties of such surfaces. We use the term "geometric techniques" to refer to the study of knot properties based on surfaces.

**Note.** In Section 4.1, we define "surface" and an "intrinsic" property of a surface. In Section 4.2 we give a classification of surfaces in terms of intrinsic properties (see Theorem 4.2.5 and 4.2.6). In Section 4.3 we consider applications of surfaces to knot theory and argue that every knot is the boundary of some surface (see Theorem 4.3.7). In Sections 4.4 we consider "surgery" on surfaces and in Section 4.5 we we develop the Prime (Knot) Decomposition Theorem (Theorem 4.5.9).

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