Section 9.2. Three Dimensions from a 2-Dimensional Perspective

Note. In this brief section, we consider 2-dimensional slices of 3-dimensional objects. The purpose is to prepare us for taking 3-dimensional slices of knots in 4-dimensions in the next section.

Note. Livingston mentions Edwin Abbott Abbott's *Flatland: A Romance of Many Dimensions*, originally published in 1884 in London. The story involves the the daily lives of two dimensional beings and their surroundings (along with a dose of social commentary). Based on our perspective as 3-dimensional beings observing these 2-dimensional creatures, we gain insight about how things might appear another dimension higher in 4-dimensions. The book is available for free download on Google Books (accessed 4/14/2021).

Note. One way to conceive of a 4-dimensional space is to use time the fourth coordinate in a 4-space of points of the form (x, y, z, t). In this way, we can think of a movie as a 4-dimensional object (well, a 3-D movie at least). We can locate points in the movie in terms of these 4 coordinates. In what follows, we consider cross sections of a 3-dimensional object by taking intersections of the object with a plane and letting the plane move through the object (in time, say). In this way we can replace a dimension (the dimension related to the vertical axis in the examples that follow) by time as the plane "moves."

Note. Consider the horizontal plane in Figure 9.2 (left). It intersects a sphere. When (there's your temporal dimension already) the plane touches the sphere at its uppermost point, the intersection is a single point. As the plane moves down, the intersection with the sphere becomes a small circle which grows in time. When the plane reaches the center of the sphere, the intersection is a circle. As the plane continues to move down, the intersection is a circle which decreases in size until the plane intersects the lowermost point on the sphere. So we get a 2-dimensional "movie" that plays through time. Figure 9.2 (right) gives five frames from that movie.

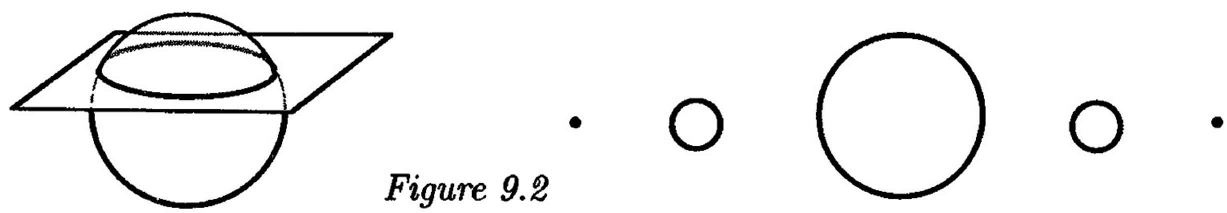

Figure 9.2

Note. Figures 9.3 and 9.4 give similar 2-dimensional movies. In both cases the plane moves vertically down through the surface and the cross sections are given. The little circles drawn in perspective on the surfaces are meant to reflect roundness of the surface (not any particular intersection with the surface).

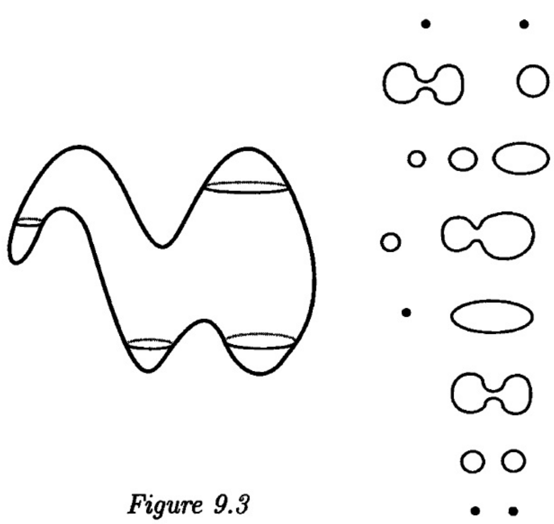

Figure 9.3

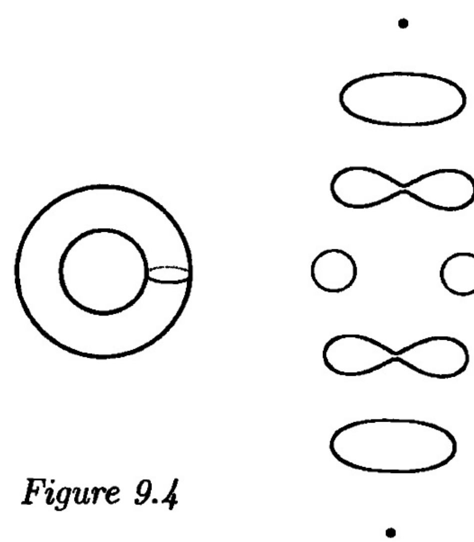

Figure 9.4

Note. Figure 9.5 gives a 2-dimensional movie of a plane moving vertically through a link (instead of a surface). Each cross section is simply finite set of points (coded by size in the figure to distinguish the individual components of the link from one another). Notice that near the center of the cross section we see two points "orbit" each other (indicated by the curved arrow) as a result of the intertwining of the two components.

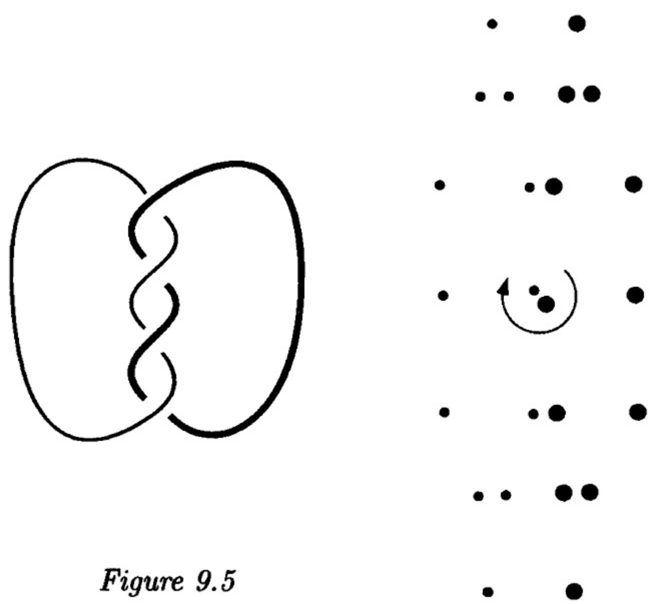

Figure 9.5

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