Chapter 10. New Combinatorial Techniques
Study Guide

The following is a brief list of topics covered in Chapter 10 of Charles Livingston's *Knot Theory*, The Carus Mathematical monographs, Volume 24 (MAA, 1993). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

10.1. The Conway Polynomial of a Knot.
Normalized Alexander polynomial, Conway polynomial, examples of Conway polynomials (Examples 10.1.A and 10.1.B), right-handed crossing L_+/left-handed crossing L_-/smoothes crossing L_S and their use in the recursive relationship for the Conway polynomial (the "skein relation" of Note 10.1.A), using the recursive relation to find the Conway polynomial (Exampled 10.1.C and 10.1.D), Conway polynomials for connected sums/mirror images/reverses of knots (Theorem 10.1.1), use of the recursive relation and Figure 10.5 (beware of the typographical errors in Livingston's book).

10.2. New Polynomial Invariants.
Jones polynomial (no details are given in the notes or book), HOMFLY polynomial and its recursive formula, finding the HOMFLY polynomial for the unlink of two components.

10.3. Kaufman's Bracket Polynomial.
Smoothing of type A, smoothing of type B, state of a smoothed diagram, the polynomial $\langle D \rangle$ for a link diagram D, computation of $\langle D \rangle$ for the trefoil knot of Figure 10.10 (beware the typographical error in Livingston), Reidemeister moves and $\langle D \rangle$, the Kaufman polynomial $F[K](t) = (-t)^w \langle D \rangle$ where w is the number of right-handed crossing minus the number of left-handed crossings in an orientation of D, the Kauffman polynomial for the trefoil and its use to find the Jones polynomial.

Revised: 4/13/2021