

# Chapter 2. What is a Knot?

## Study Guide

The following is a brief list of topics covered in Chapter 2 of Charles Livingston's *Knot Theory*, The Carus Mathematical monographs, Volume 24 (MAA, 1993). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

### 2.1. Wild Knots and Unknottings.

A knot as the image of  $[0, 1]$  in 3-space under a continuous function, wild knot (a knot with an infinite number of crossings), tame knot, using differentiability to deal with problems of continuity (see Figure 2.3), "Differentiable Knots are Tame" in Crowell and Fox.

### 2.2. The Definition of a Knot.

Closed polygonal curve in 3-space, simple curves, definition of a knot as a simple closed polygonal curve in 3-space, vertices of a knot, definition of a link as a finite union of disjoint knots, unlink.

### 2.3. The Equivalence of Knots, Deformations.

Elementary deformation, equivalent knots.

### 2.4. Diagrams and Projections.

Projection map, projection of a knot, diagram of a knot, knots with the same diagram are equivalent, regular projection, small changes in vertices of a knot can produce an equivalent regular knot (Theorem 2.4.1), small changes in vertices of a regular knot can produce an equivalent regular knot (Theorem 2.4.2), two knots with regular projections and identical diagrams are equivalent (Theorem 2.4.3), edges/arcs in a knot diagram, crossings/crossing points in a knot diagram.

### 2.5. Orientations.

Using  $n$ -tuples of vertices to put a "direction" on a knot, oriented knot, elementary deformations of oriented knots produce equivalent oriented knots, oriented equivalent knots, reverse of an oriented knot, reversible knots, non-reversible knots exist (Trotter's 1963 result about  $(p, q, r)$ -pretzels.

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