

Chapter 3. Combinatorial Techniques

Study Guide

The following is a brief list of topics covered in Chapter 3 of Charles Livingston's *Knot Theory*, The Carus Mathematical monographs, Volume 24 (MAA, 1993). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

3.1. Reidemeister Moves.

The six Reidemeister moves, equivalent knots have diagrams related by Reidemeister moves (Theorem 3.1.1).

3.2. Colorings.

Colorable knot diagram, colorability of all diagrams of a knot (Theorem 3.2.2), colorable knot.

3.3. A Generalization of Colorability, mod p Labelings.

A mod p labeling of a knot diagram, mod p labeling of all diagrams of a knot (Theorem 3.3.3).

3.4. Matrices, Labelings, and Determinants.

Introducing a system of equations (modulo p) related to a mod p labeling, the augmented matrix for such a system of equations, homogeneous systems over a field, finite fields, row reduction and the creation of an $(n-1) \times (n-1)$ matrix from the $n \times n$ coefficient matrix (Note 3.4.A), relating a mod p labeling to a matrix (Theorem 3.4.4), determinant of a knot, mod p rank of a knot, diagram independence of determinant and mod p rank (Theorem 3.4.5).

3.5. The Alexander Polynomial.

Left-hand and right-hand crossings in an oriented knot, creation of an $n \times n$ matrix (Note 3.5.A), the $(n-1) \times (n-1)$ Alexander matrix, Alexander polynomial, the relationship between Alexander polynomials for different oriented labeled diagrams of a given knot (Theorem 3.5.6), "the" Alexander polynomial versus "an" Alexander polynomial of a knot (Note 3.5.B), computation of Alexander polynomials (Examples 3.5.1 and 3.5.2), the $(2, n)$ -torus knots form an infinite family of knots.