

Chapter 4. Geometric Techniques

Study Guide

The following is a brief list of topics covered in Chapter 4 of Charles Livingston's *Knot Theory*, The Carus Mathematical monographs, Volume 24 (MAA, 1993). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

4.1. Surfaces and Homeomorphisms.

Triangle, polyhedral surface, boundary of a surface, triangulation of a surface, orientable surface, nonorientable surface/Möbius strip, intrinsic properties, homeomorphic properties, extrinsic properties.

4.2. The Classification of Surfaces.

Informal idea of the genus of a surface, the different versions of the Classification Theorem, Euler characteristic of a surface, genus of a surface, Euler characteristic of a union of surfaces (Theorem 4.2.1), genus of a union of surfaces (Corollary 4.2.2), the genus of a surface that results by adding bands to disks (Theorem 4.2.3), Euler characteristic of the union of surfaces that intersect in circles (Theorem 4.2.4), Classification Theorem I (Theorem 4.2.5), Classification Theorem II (Theorem 4.2.6).

4.3. Seifert Surfaces and the Genus of a Knot.

Every knot is the boundary of an orientable surface (Theorem 4.3.7), Seifert surface, genus of a knot, genus of trefoil knot is 1.

4.4. Surgery on Surfaces.

Puncturing a surface, surgery on a surface along a disk, a separating boundary of a disk, the effect on the genus of a surface by surgery (Theorem 4.4.8).

4.5. Connected Sums on Knots and Prime Decompositions.

Connected sum $K_1 \# K_2$, prime knot, Prime Decomposition Theorem (Theorem 4.5.9), references for proofs of the Prime Decomposition Theorem, Additivity of Knot Genus (Theorem 4.5.10), cut-and-paste proof (see the proof of Theorem 4.5.10).

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