Chapter 5. Algebraic Techniques
Study Guide

The following is a brief list of topics covered in Chapter 5 of Charles Livingston's *Knot Theory*, The Carus Mathematical monographs, Volume 24 (MAA, 1993). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

5.1. Symmetric Groups.
Permutation on a set, left to right reading of function composition and cycle multiplication, symmetric group S_n, group (binary operation, associativity, identity, inverse), cyclic notation, a set that generates a group, transposition.

5.2. Knots and Groups.
Labeling of an oriented knot diagram with elements of a group, consistency, generation, a labeling with elements of S_4 exists if and only if a labeling with elements of S_3 exists Perko's 1975 result), different diagrams for the same knot can can always be labeled by elements from the same group (Theorem 5.2.1).

5.3. Conjugation and the Labeling Theorem.
Cycle structure of a permutation in S_n, conjugate elements/conjugacy class, two elements of S_n are conjugate if and only if they have the same cycle structure (Theorem 5.3.A), labeling oriented knots withe elements from a conjugacy class (Theorem 5.3.2) and the use to distinguish knots (6_1 and 9_{46} in the notes).

5.4. Equations in Groups and the Group of a Knot.
Using conjugacy relations to find a labeling of a knot diagram, generators/relations/group presentations, the group of a knot resulting from a labeling, the group of a knot is an invariant.

5.5. The Fundamental Group.
Base point p, closed oriented paths that begin and end at p, homotopic paths, homotopy class, fundamental group, the binary operation/identity/inverses in the fundamental group, the group of a knot resulting from a labeling is isomorphic to the fundamental group of the knot (Note 5.5.A), labelings of a knot diagram correspond to homomorphisms of the fundamental group (Note 5.5.C).

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