Chapter 6. Geometry, Algebra, and the Alexander Polynomial

Study Guide

The following is a brief list of topics covered in Chapter 6 of Charles Livingston's *Knot Theory*, The Carus Mathematical monographs, Volume 24 (MAA, 1993). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

6.1. The Seifert Matrix.
Disks with bands added, signs of crossings, linking number, positive push off of a curve on an orientable surface, the number of bands is twice the genus (Note 6.1.A), Seifert matrix, computing linking numbers and Seifert matrices.

6.2. Seifert Matrices and the Alexander Polynomial.
Computing the Alexander polynomial using the Seifert matrix (Theorem 6.2.1), relationship between $A_K(t)$ and $A_K(t^{-1})$ (Corollary 6.2.2), band move on a Seifert surface, adding two bands to a disk as stabilization, S-equivalence of matrices, two Seifert matrices for a given knot are S-equivalent (Theorem 6.2.3).

6.3. The Signature of a Knot, and the other S-Equivalence Invariants.
Diagonalizability of real symmetric matrices, signature of a real symmetric matrix, signature of a knot, Sylvester's Law of Inertia, the signature of a knot is a knot invariant (Theorem 6.3.5), Hermitian matrices, ω-signature of a knot where $\omega \in \mathbb{C}$ and $|\omega| = 1$, the signature function of a knot.

6.4. Knot Groups and the Alexander Polynomial.
Free differential calculus/Fox calculus, a word in terms of variables, the Fox derivative of a word, computation of Fox derivatives (Example 6.4.A), using the Fox derivative to find the Alexander polynomial of the trefoil knot, the Jacobian matrix of a vector function, Fox's Algorithm (Note 6.4.A), applying Fox's Algorithm to find the Alexander polynomial of the trefoil knot (Example 6.4.B), computations which use relations from the group of a knot to find the matrix $A(t)$ and ultimately the Alexander polynomial (Example 6.4.C).