Chapter 7. Numerical Invariants
Study Guide

The following is a brief list of topics covered in Chapter 7 of Charles Livingston's *Knot Theory*, The Carus Mathematical monographs, Volume 24 (MAA, 1993). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

7.1. Summary of Numerical Invariants.
Review of numerical knot invariants: genus, mod p rank, determinant, signature, degree of the Alexander polynomial.

7.2. New Invariants.
The crossing index of a knot, crossing changes, unknotting number, an algorithm to convert a knot into the unknot using crossing changes (see Exercise 7.2.3), affect on the unknotting number by taking connected sums (conjectured to be additive), Bleiler's "fascinating example."

7.3. Braids and Bridges.
n-stranded braid, closed braid, Alexander's technique of showing that every knot and link arises from a braid and its application (see Exercise 7.3.2(a)), braid index, subadditivity of the braid index, braid groups and the binary operation on n-stranded braids, stabilization/Markov move M_2, conjugation/Markov move M_1, Markov equivalent braids, relative maxima of a knot diagram, bridge index, the bridge index and knot sums (Theorem 7.3.1).

7.4. Relations Between the Numerical Invariants.
The number of Seifert circles s genus g and crossing number cr are related as $2g = cr - s + 1$, mod p rank is at most the bridge index minus 1, unknotting number $U(K)$ and signature $\sigma(K)$ are related as $2U(K) \geq |\sigma(K)|$, the unknotting number is greater than or equal to the mod p rank.

7.5. Independence of Numerical Invariants.
The bridge index is unrelated to the degree of the Alexander polynomial, the mod p rank can vary for different values of p, no bound on the signature can be based on the bridge index, the bridge index can be 2 while the unknotting number can be arbitrarily large (for example, for the $(2,n)$-torus knot), transpositions and the bridge index (Theorem 7.2), there exist doubled knots of large bridge index (Notes 7.5.A and 7.5.B), no bound on the genus can be based on thebridge index (for example, for the $(2,n)$-torus knot).

Revised: 3/16/2021