Chapter 9. High-Dimensional Knot Theory

Study Guide

The following is a brief list of topics covered in Chapter 9 of Charles Livingston's *Knot Theory*, The Carus Mathematical monographs, Volume 24 (MAA, 1993). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

9.1. Defining High-Dimensional Knots.
A k-sphere in $\mathbb{R}^{k+1}$, the "natural relationship" between points and vectors; standard position, surfaces as 2-manifolds, smooth knotted k-sphere, differentiable function $F : S^k \to \mathbb{R}^n$, smoothly equivalent k-knots in $\mathbb{R}^n$.

9.2. Three Dimensions from a 2-Dimensional Perspective.
Flatland by E. A. Abbott (1884), interpretation of 4-space as space-time (x, y, z, t) and a 4-dimensional object as a "movie," a sequence of 3-D cross sections of 2-knots (see Figures 9.3, 9.4, and 9.5).

9.3. Three-Dimensional Cross-Sections of a 4-Dimensional Knot.
More sequences of 3-D cross sections of 2-knots, the spun trefoil (Figure 9.8), k-twist spin trefoil (see Figure 9.9), band moves, a colorable 4-knot.

9.4. Slice Knots.
Slice knot, hyperplane of 4-space, smooth disk, slice disk, ribbon knot, every ribbon knot is slice (Theorem 9.4.1), the conjecture that every slice knot is ribbon, factorization of the Alexander polynomial of a slice knot (Corollary 9.4.3), the signature of a slice knot is 0 (Corollary 9.4.4), algebraic slice.

9.5. The Knot Concordance Group.
Concordant knots, concordance as an equivalence relation (Theorem 9.5.5), concordance and connected sums (Lemma 9.5.6), the abelian group C_1^3 (see Theorem 9.5.7), properties of C_1^3.

Revised: 4/24/2021