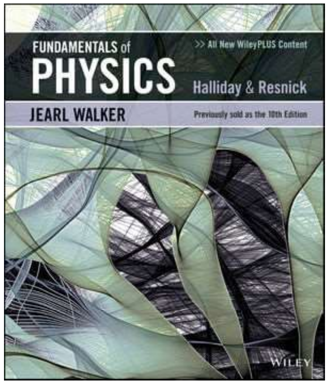
Technical Physics 1—Calculus Based

Chapter 2. Motion Along a Straight Line—Examples, Questions, Problems



Problem 2.6

Problem 2.6. The 1992 world speed record for a bicycle (human-powered vehicle) was set by Chris Huber. His time through the measured 200 m stretch was a sizzling 6.509 s. In 2001, Sam Whittingham beat Huber's record by 19.0 km/h. What was Whittingham's time through the 200 m?

Solution. First, the total distance is 200 m. Since Huber's time is 6.509 s, then formula (2-3) for average speed gives us that Huber's average speed is (using four significant figures, as with the value of time):

$$s_{\text{avg}} = \frac{\text{total distance}}{\Delta t} = \frac{200 \text{ m}}{6.509 \text{ s}} \approx 30.73 \text{ m/s}.$$

Notice that we can use two conversion factors to express 19.0 km/h in terms of m/s (using three significant figures as with 19.0 km/h):

$$19.0 \text{ km/h} = \frac{19.0 \text{ km}}{1 \text{ h}} \times 1000 \text{ m/km} \times \frac{1 \text{ h}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}}.$$

Problem 2.6 (continued)

Problem 2.6. The 1992 world speed record for a bicycle (human-powered vehicle) was set by Chris Huber. His time through the measured 200 m stretch was a sizzling 6.509 s. In 2001, Sam Whittingham beat Huber's record by 19.0 km/h. What was Whittingham's time through the 200 m?

Solution (continued).

$$19.0 \text{ km/h} = \frac{19000 \text{ m}}{1 \text{ h}} \times \frac{1 \text{ h}}{3600 \text{ sec}} \approx 5.28 \text{ m/s}.$$

So Whittingham's average speed was Huber's average speed plus 19.0 km/h: $30.73 \text{ m/s} + 19.0 \text{ km/h} = 30.73 \text{ m/s} + 5.28 \text{ m/s} = 36.01 \text{ m/s}$. Therefore Whittingham's time (also from formula (2-3), rearranged) was

$$\Delta t = \frac{\text{total distance}}{\text{average speed}} \approx \frac{200 \text{ m}}{36.01 \text{ m/s}} \approx \boxed{5.554 \text{ s}}.$$

□

Problem 2.16

Problem 2.16. The position function $x(t)$ of a particle moving along an x axis is $x = 4.0 - 6.0t^2$, with x in meters and t in seconds. **(a)** At what time and **(b)** where does the particle (momentarily) stop? At what **(c)** negative time and **(d)** positive time does the particle pass through the origin? **(e)** Graph x versus t for the range -5 s to $+5$ s. **(f)** To shift the curve rightward on the graph, should we replace the term 't' with $t + 20$ or $t - 20$ in $x(t)$? **(g)** Does that replacement increase or decrease the value of x at which the particle momentarily stops?

Solution. First, the velocity of the particle, by formula (2-4), is $v(t) = dx/dt = -12.0t$ m/s.

(a, b) The particle "momentarily stops" when the velocity is 0 m/s. So we set $v(t) := 0$ and solve: $v(t) = -12.0t \text{ m/s} := 0 \text{ m/s}$ to get $\boxed{t = 0 \text{ s}}$. At time $t = 0$ s, we have the position of the particle as $x = 4.0 - 6.0(0)^2 \text{ m} = \boxed{4.0 \text{ m}}$.

Problem 2.16 (continued 1)

Problem 2.16. The position function $x(t)$ of a particle moving along an x axis is $x = 4.0 - 6.0t^2$, with x in meters and t in seconds. At what **(c)** negative time and **(d)** positive time does the particle pass through the origin? **(e)** Graph x versus t for the range -5 s to $+5$ s. **(f)** To shift the curve rightward on the graph, should we replace the term 't' with $t + 20$ or $t - 20$ in $x(t)$? **(g)** Does that replacement increase or decrease the value of x at which the particle momentarily stops?

Solution (continued). (c, d) The "origin" on the number line corresponds to $x = 0$, so we set $x = 4.0 - 6.0t^2$ m $:= 0$ m to get $t^2 = 4.0/6.0$ s, or $\sqrt{t^2} = \sqrt{2.0/3.0}$ s, or $|t| = \sqrt{2.3/3.0}$ s. Then (to two significant figures, like position) $t = -\sqrt{2.0/3.0} \approx -0.82$ s and $t = \sqrt{2.0/3.0} \approx 0.82$ s. That is, the desired negative time is $\boxed{-0.82 \text{ s}}$ and the desired positive time is $\boxed{0.82 \text{ s}}$.

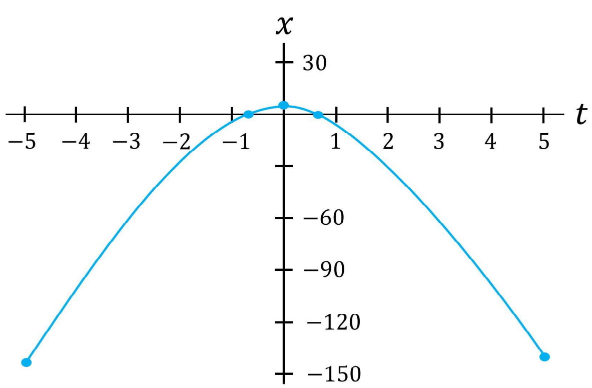
Problem 2.16 (continued 2)

Problem 2.16. The position function $x(t)$ of a particle moving along an x axis is $x = 4.0 - 6.0t^2$, with x in meters and t in seconds. **(e)** Graph x versus t for the range -5 s to $+5$ s. **(f)** To shift the curve rightward on the graph, should we replace the term 't' with $t + 20$ or $t - 20$ in $x(t)$? **(g)** Does that replacement increase or decrease the value of x at which the particle momentarily stops?

Solution (continued). We know that the graph of $x(t) = 4.0 - 6.0t^2$ is a concave down parabola in the (t, x)-plane with a vertex at $(t, x) = (0, 4.0)$ by parts (a) and (b); the vertex corresponds to an absolute maximum of the graph by the First Derivative Test (see my online notes for Calculus 1 [MATH 1910] on Section 4.3. Monotonic Functions and the First Derivative Test). By parts (c) and (d) we know that the graph has intercepts of $(-0.82, 0)$ and $(0.82, 0)$. When $t = -5$ s, we have $x(-5) = 4.0 - 6.0(-5)^2 = 4.0 - 6.0(25) = -146.0$ m. When $t = +5$ s, we have $x(+5) = 4.0 - 6.0(+5)^2 = 4.0 - 6.0(25) = -146.0$ m.

Problem 2.16 (continued 3)

Problem 2.16. The position function $x(t)$ of a particle moving along an x axis is $x = 4.0 - 6.0t^2$, with x in meters and t in seconds. **(e)** Graph x versus t for the range -5 s to $+5$ s.
Solution (continued). Based on the vertex, intercepts, the points $(-5, -146.0)$ and $(+5, 146.0)$, and the known shape of the curve we get:



Problem 2.16 (continued 4)

Problem 2.16. The position function $x(t)$ of a particle moving along an x axis is $x = 4.0 - 6.0t^2$, with x in meters and t in seconds. **(f)** To shift the curve rightward on the graph, should we replace the term 't' with $t + 20$ or $t - 20$ in $x(t)$? **(g)** Does that replacement increase or decrease the value of x at which the particle momentarily stops?
Solution (continued). (f) To shift the curve rightward 20 units, we want the vertex of the parabola shifted from $t = 0$ to $t = 20$. We $\boxed{\text{replace } t \text{ with } t - 20}$ to get $x(t) = 4.0 - 6.0(t - 20)^2$. Notice that this new function implies a velocity of $v(t) = dx/dt = -12.0(t - 20)$ so that velocity is 0 m/s with $t = 20s$ instead of $t = 0$ s, as desired. That is, that replacement $\boxed{\text{increases the value of } x}$ at which the particle momentarily stops. This is also how one performs a horizontal shift of a graph to the right; see my online notes for Precalculus-Algebra (MATH 1710) on Section 2.5. Graphing Techniques: Transformations.

Problem 2.16 (continued 5)

Note. Problem 2.16(f) as stated in the book asks: To shift the curve rightward on the graph, should we include the term $+20t$ or the term $-20t$ in $x(t) = 4.0 - 6.0t^2$? By adding such terms (or "including" them), we get the new position functions

$$x_1(t) = 4.0 - 6.0t^2 - 20t \text{ and } x_2(t) = 4.0 - 6.0t^2 + 20t.$$

However, these are not just shifts of the curve leftward or rightward, but also involves a vertical shift and possible compressions or expansions. Nonetheless, these position functions have corresponding velocity functions $v_1(t) = -12.0t - 20$ and $v_2(t) = -12.0t + 20$, which give zero velocity at $t_1 = -20/12.0 \approx -1.67$ s and $t_2 = 20/12.0 \approx 1.67$ s, respectively. For position function $x_1(t)$, the vertex has been shifted leftward (*and downward*; WHY?). For position function $x_2(t)$, the vertex has been shifted rightward (*and upward*; WHY?). Of course the graph is still a concave down parabola, but it is not the same size as the parabola that is given by graphing $x(t)$ in part (f). □

Problem 2.20

Problem 2.20. (a) If the position of a particle is given by $x = 20t - 5t^3$, where x is in meters and t is in seconds, when, if ever, is the particle's velocity zero? **(b)** When is its acceleration a zero? **(c)** For what time range (positive or negative) is a negative? **(d)** Positive? **(e)** Graph $x(t)$, $v(t)$, and $a(t)$.

Solution. First, with $x(t) = 20t - 5t^3$, we have by formula (2-4) we have velocity $v(t) = 20 - 15t^2$ and by formula (2-8) we have acceleration $a(t) = -30t$.

(a) For velocity to be 0 m/s, we need $v(t) = 20 - 15t^2 := 0$ m/s, or $t^2 = 20/15 = 4/3$ s^2, or $t = \pm\sqrt{4/3}$ s. That is, the velocity is 0 m/s when $\boxed{t = -\sqrt{4/3} = -2/\sqrt{3} \approx -1.15470}$ and when $\boxed{t = \sqrt{4/3} = 2/\sqrt{3} \approx 1.15470}$.

(b) Acceleration is 0 m/s^2 when $a(t) = -30t := 0$ m/s^2, or when $\boxed{t = 0 \text{ s}}$.

Problem 2.20 (continued 1)

Problem 2.20. (a) If the position of a particle is given by $x = 20t - 5t^3$, where x is in meters and t is in seconds, when, if ever, is the particle's velocity zero? **(b)** When is its acceleration a zero? **(c)** For what time range (positive or negative) is a negative? **(d)** Positive? **(e)** Graph $x(t)$, $v(t)$, and $a(t)$.

Solution (continued). (c, d) To determine when acceleration $a(t)$ is positive or negative, we perform a sign test on it. We know that $a(t) = -30t$ is a continuous function (it is a polynomial; its graph is a line), so if it changes sign then it must pass through zero to do so (this is the Intermediate Value Theorem; see my online Calculus 1 [MATH 1910] notes on Section 2.5. Continuity and notice Theorem 2.11). Since acceleration is only 0 m/s^2, the $a(t) = -30t$ has the same sign for all $t < 0$ s, and has the same sign for all $t > 0$ s. Clearly, $\boxed{\text{acceleration is negative for } t > 0 \text{ s}}$, and it is $\boxed{\text{positive for } t < 0 \text{ s}}$.

Problem 2.20 (continued 2)

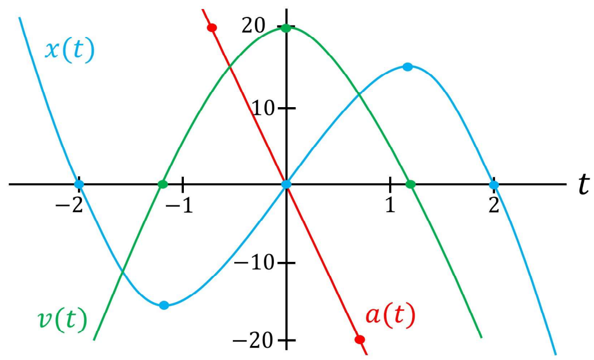
Problem 2.20. (e) Graph $x(t) = 20t - 5t^3$, $v(t) = 20 - 15t^2$, and $a(t) = -30t$.

Solution (continued). (e) We see that $x(t) = 20t - 5t^3 = 5t(4 - t^2)$ $= 5t(2 - t)(2 + t)$ has t-intercepts of $t = \pm 2$ s, and $t = 0$ s. Since $dx/dt = v(t) = 0$ m/s when $t = \pm 2/\sqrt{3} \approx \pm 1.15470$ s by part (a), then these correspond to possible extrema of $x(t)$ (this is the First Derivative Test from Calculus 1; see Section 4.3. Monotonic Functions and the First Derivative Test). In fact, $x(-2/\sqrt{3}) \approx -15.39601$ and $x(2/\sqrt{3}) \approx 15.39601$. Since $d^2x/dt^2 = a(t) < 0$ for $t > 0$, and $d^2x/dt^2 = a(t) > 0$ for $t < 0$, then $x(t)$ is concave down for $t > 0$ and concave up for $t < 0$ (this is the Second Derivative Test for Concavity from Calculus 1; see Section 4.4. Concavity and Curve Sketching and Theorem 4.4.A). The velocity $v(t) = 20 - 15t^2$ has a graph that is a concave down parabola. We saw in part (a) that velocity has t-intercepts at $t = \pm 2/\sqrt{3} \approx \pm 1.15470$ s. Since $dv/dt = a(t) = 0$ when $t = 0$ s, then the vertex of $v(t)$ occurs at point $(0, 20)$.

Problem 2.20 (continued 3)

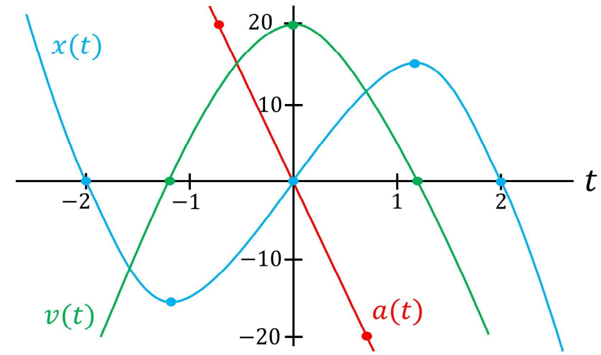
Problem 2.20. (e) Graph $x(t) = 20t - 5t^3$, $v(t) = 20 - 15t^2$, and $a(t) = -30t$.

Solution (continued). The information from the previous slide gives us:



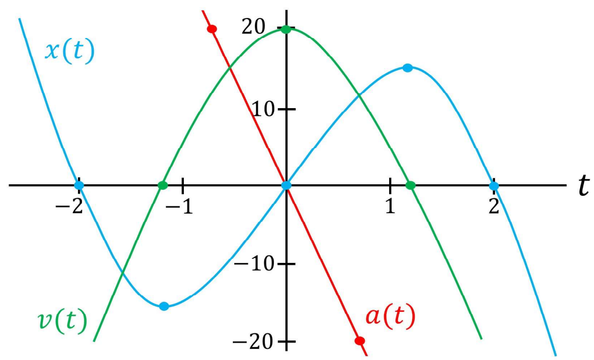
Problem 2.20 (continued 4)

Note. In terms of calculus, when $x(t)$ is decreasing, its derivative $v(t)$ is negative. When $x(t)$ is increasing, its derivative $v(t)$ is positive. Also, $x(t)$ has a local minimum when its derivative $v(t)$ is zero and its second derivative is positive, and $x(t)$ has a local maximum when its derivative $v(t)$ is zero and its second derivative is negative.



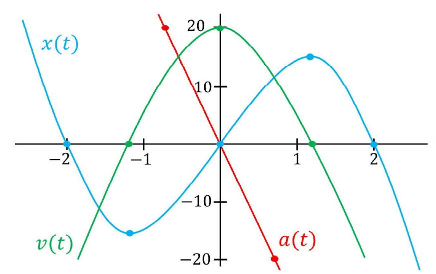
Problem 2.20 (continued 5)

Note. In terms of physics, the particle is moving backward (i.e., when $x(t)$ is decreasing), its velocity $v(t)$ is negative. When the particle if moving forward (i.e., when $x(t)$ is increasing), its velocity $v(t)$ is positive.



Problem 2.20 (continued 6)

Note (continued). Also, the particle reaches its left-most point locally (i.e., $x(t)$ has a local minimum) when its velocity $v(t)$ is zero and its acceleration is positive (positive acceleration will increase velocity making it positive, so that the particle then starts to move to the right), and the particle reaches its right-most point locally (i.e., $x(t)$ has a local maximum) when its velocity $v(t)$ is zero and its acceleration is negative (negative acceleration will decrease velocity making it negative, so that the particle then starts to move to the left).



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Problem 2.28

Problem 2.28. On a dry road, a car with good tires may be able to brake with a constant deceleration of 4.92 m/s^2. **(a)** How long does such a car, initially traveling at 24.6 m/s, take to stop? **(b)** How far does it travel in this time? **(c)** Graph x versus t and v versus t for the deceleration.

Solution. We view the car as moving along the real number line (i.e., the x-axis). The the constant acceleration is $a = -4.92$ m/s^2 (negative because the car is decelerating).

(a) The initial velocity is $v_0 = 24.6$ m/s. When the car comes to a stop, we have $v = 0$ m/s. So by equation (2-11), we have $v = v_0 + at$, or $0 = 24.6 + (-4.92)t$ m/s, and so $t = 24.6/4.92 = 5.00$ s. That is, it takes $\boxed{\text{5.00 s for the car to stop}}$.

Problem 2.28 (continued 1)

Problem 2.28. On a dry road, a car with good tires may be able to brake with a constant deceleration of 4.92 m/s^2. **(a)** How long does such a car, initially traveling at 24.6 m/s, take to stop? **(b)** How far does it travel in this time? **(c)** Graph x versus t and v versus t for the deceleration.

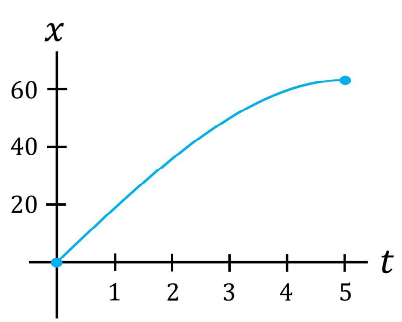
Solution (continued). (b) By equation (2-15) we have $x - x_0 = v_0 t + \frac{1}{2}at^2$, where x_0 is the initial position. We take this to be $x_0 = 0$ m (for simplicity). Then the car stops when at position $x - x_0 = x = (24.6)(5.00) + \frac{1}{2}(-4.92)(5.00)^2 = 123 - 61.5 = 61.5$ m. So the $\boxed{\text{car has traveled 61.5 m}}$ in the time before it stops.

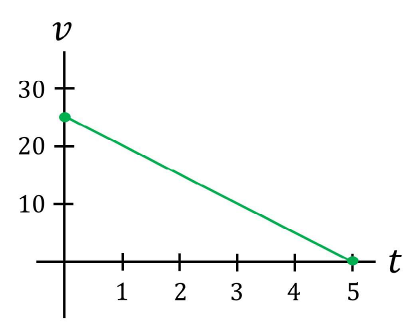
(c) By part (b), with $x(t) = v_0 t + \frac{1}{2}at^2 = 24.6t + \frac{1}{2}(-4.92)t^2 = 24.6t - 2.46t^2$, we see that the graph is a concave down parabola. By part (a), $v(t) = v_0 + at = 24.6 + (-4.92)t = 24.6 - 4.92t$ m/s. Since the velocity is 0 m/s when $t = 5$ s, and $v(t)$ is the derivative of $x(t)$, then the graph of $x(t)$ has its vertex at $t = 5$ s. So we graph $x(t)$ and $v(t)$ for $0 \le t \le 5$.

Problem 2.28 (continued 2)

Problem 2.28. (c) Graph $x(t) = 24.6t - 2.46t^2$ m, and $v(t) = 24.6 - 4.92t$ m/s, for the deceleration.

Solution (continued).





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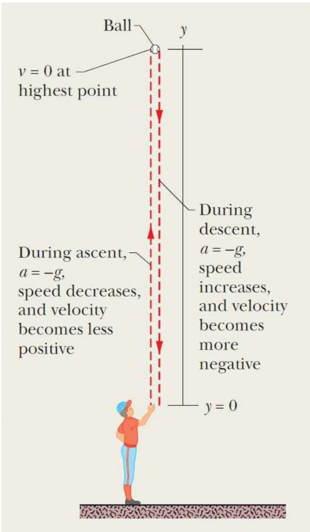
Sample Problem 2.05

Sample Problem 2.05. A pitcher tosses a baseball up along a y axis with an initial speed of 12 m/s. See Figure 2-13. **(a)** How long does the ball take to reach its maximum height? **(b)** What is the ball's maximum height above its release point? **(c)** How long does the ball take to reach a point 5.0 m above its release point?



Solution. As given in the diagram, we have $y = 0$ m as the height at which the ball is released. The initial velocity is given as $v_0 = 12$ m/s (positive, since the ball goes up). The constant acceleration due to gravity is $g = -9.8$ m/s. So the height is given by $y = \frac{1}{2}(-9.8)t^2 + v_0 t + y_0$ or $y = -4.9t^2 + 12t$ m.

Sample Problem 2.05 (continued 1)

Sample Problem 2.05. A pitcher tosses a baseball up along a y axis with an initial speed of 12 m/s. See Figure 2-13. **(a)** How long does the ball take to reach its maximum height? **(b)** What is the ball's maximum height above its release point? **(c)** How long does the ball take to reach a point 5.0 m above its release point?

Solution (continued). (a) By differentiation, the velocity function is $dy/dt = -9.8t + 12$ m/s, so the velocity is 0 m/s when $0 = -9.8t + 12$ m/s, or when $t = 12/9.8 \approx 1.2$ s. When velocity is 0 m/s, the ball is at its maximum height, the the maximum height occurs when $\boxed{t = 1.2 \text{ s}}$.

(b) When $t = 1.2$ s, the height is $y = -4.9(1.2)^2 + 12(1.2) \approx 7.3$ m. That is, the ball's $\boxed{\text{maximum height is } y = 7.3 \text{ m}}$.

Sample Problem 2.05 (continued 2)

Sample Problem 2.05. A pitcher tosses a baseball up along a y axis with an initial speed of 12 m/s. See Figure 2-13. **(a)** How long does the ball take to reach its maximum height? **(b)** What is the ball's maximum height above its release point? **(c)** How long does the ball take to reach a point 5.0 m above its release point?

Solution (continued). (c) To find when height is 5 m, we set $y = -4.9t^2 + 12t := 5$ m, or $-4.9t^2 + 12t - 5 = 0$ m. From the quadratic formula, we then have

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(12) \pm \sqrt{(12)^2 - 4(-4.9)(-5)}}{2(-4.9)}$$

$$= \frac{-12 \pm \sqrt{46}}{-9.8} \approx \frac{-12 \pm 6.78}{-9.8}.$$

So the ball has height 5 m (to two significant figures) when $\boxed{t = 0.53 \text{ s}}$ (when it is going up) and $\boxed{t = 1.9 \text{ s}}$ (when it is falling down. □

Problem 2.66

Problem 2.66. In a forward punch in karate, the fist begins at rest at the waist and is brought rapidly forward until the arm is fully extended. The speed $v(t)$ of the fist is given in Figure 2-37 for someone skilled in karate. The vertical scaling is set by $v_s = 8.0$ m/s. How far has the fist moved at **(a)** time $t = 50$ ms and **(b)** when the speed of the fist is maximum?

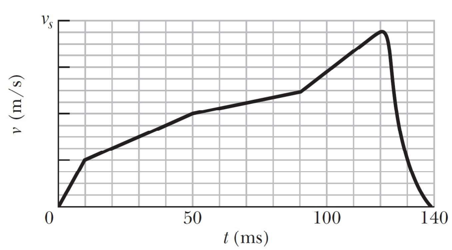
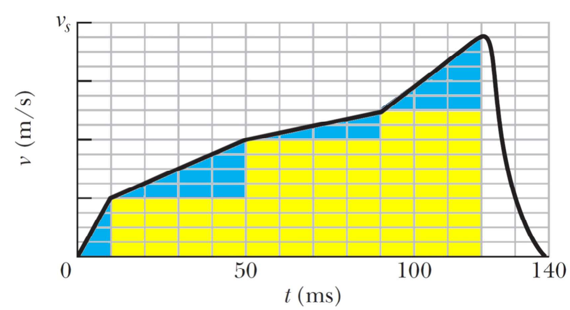


Figure 2-37 Problem 66.

Solution. By equation (2-30), we know the change in position (i.e., the distance moved) is the area between the velocity curve and the time axis over the time interval.

Problem 2.66 (continued 1)

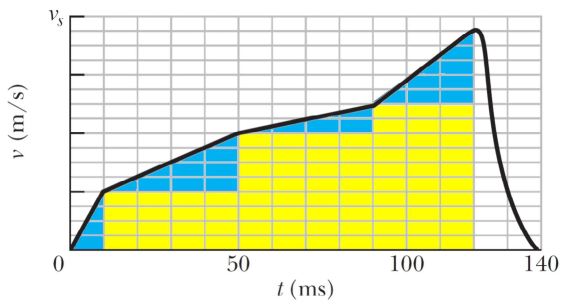
Solution (continued).



Since the vertical scaling is at $v_s = 8.0$ m/s, so each little grey box has height $8.0/16 = 0.5$ m/s. We convert this to the units of m/ms (i.e., meters per millisecond) with the conversion factor 1/1000 s/ms to get (0.5 m/s)(1/1000 s/ms) = 0.0005 m/ms.

Problem 2.66 (continued 2)

Solution (continued). (a)

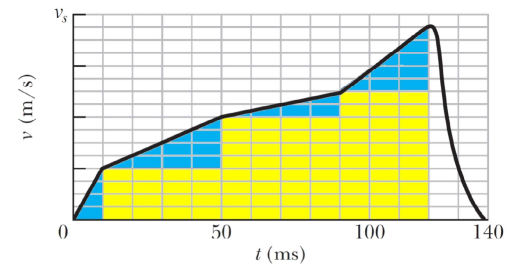


We need the area under the curve for $0 \leq t \leq 50$ ms. We introduce rectangles (in yellow) and triangles (in blue), as given above. So the area under the curve for $0 \leq t \leq 50$ ms is
$\frac{1}{2}(10 \text{ ms})(0.0020 \text{ m/ms}) + (40 \text{ ms})(0.0020 \text{ m/ms}) + \frac{1}{2}(40 \text{ ms})(0.0020 \text{ m/ms}) = 0.01 + 0.080 + 0.040 = \boxed{0.13 \text{ m}}$.

Problem 2.66 (continued 3)

Problem 2.66. How far has the fist moved **(b)** when the speed of the fist is maximum?

Solution (continued). (b)



The fist has maximum speed at $t = 120$ ms, so we need the area under the curve for $0 \leq t \leq 120$ ms. Using the answer to part (a), we have the area $0.13 \text{ m} + (40 \text{ ms})(0.0040 \text{ m/ms}) + \frac{1}{2}(40 \text{ ms})(0.0010 \text{ m/ms}) + (30 \text{ ms})(0.0050 \text{ m/ms}) + \frac{1}{2}(30 \text{ ms})(0.0025 \text{ m/ms}) = 0.13 + 0.16 + 0.020 + 0.15 + 0.038 = \boxed{0.50 \text{ m}}$. □