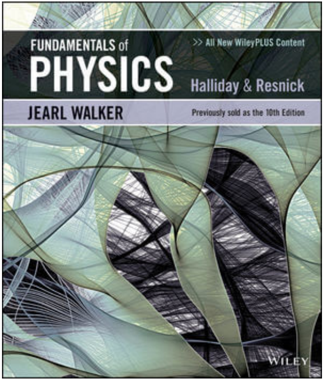
# Technical Physics 1—Calculus Based

**Chapter 3. Vectors**—Examples, Questions, Problems



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# Question 3.2

**Question 3.2.** The two vectors shown in Figure 3-21 lie in an *xy*-plane. What are the signs of the $x$ and $y$ components, respectively, of **(a)** $\vec{d}_1 + \vec{d}_2$, **(b)** $\vec{d}_1 - \vec{d}_2$, and **(c)** $\vec{d}_2 - \vec{d}_1$?

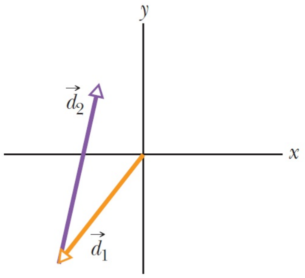


**Figure 3-21** Question 2.

**Solution.** Adding vectors as given in Figure 3-2(b), and subtracting vectors as given in Figure 3-6(b), we get the following graphs.

# Question 3.2

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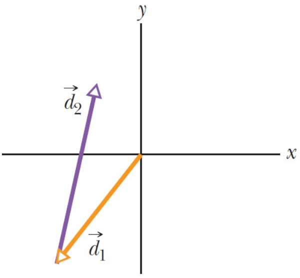


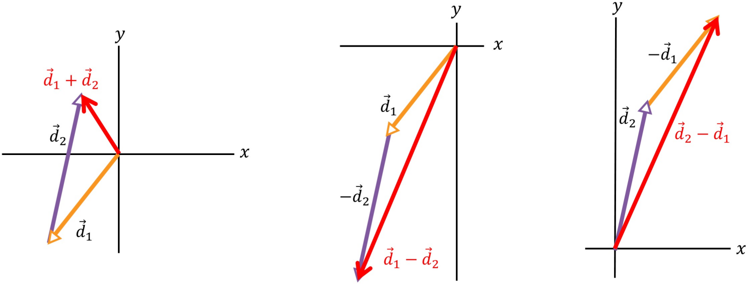

**Figure 3-21** Question 2.

**Solution.** Adding vectors as given in Figure 3-2(b), and subtracting vectors as given in Figure 3-6(b), we get the following graphs.

# Question 3.2 (continued)

**Question 3.2.** The two vectors shown in Figure 3-21 lie in an $xy$-plane. What are the signs of the $x$ and $y$ components, respectively, of **(a)** $\vec{d}_1 + \vec{d}_2$, **(b)** $\vec{d}_1 - \vec{d}_2$, and **(c)** $\vec{d}_2 - \vec{d}_1$?

**Solution (continued).**




Therefore, the $x$ and $y$ components **(a)** of $\vec{d}_1 + \vec{d}_2$ are negative and positive, respectively, **(b)** of $\vec{d}_1 - \vec{d}_2$ are both negative, and **(c)** of $\vec{d}_2 - \vec{d}_1$ are both positive. □

# Question 3.6

**Question 3.6.** Find two vectors $\vec{a}$ and $\vec{b}$ in $\mathbb{R}^2$ (and their components) such that **(a)** $\vec{a}+\vec{b}=\vec{c}$ and $a+b=c$; **(b)** $\vec{a}+\vec{b}=\vec{a}-\vec{b}$; **(c)** $\vec{a}+\vec{b}=\vec{c}$ and $a^2+b^2=c^2$.

**Solution.** **(a)** In words, we want the magnitude of $\vec{a}+\vec{b}$ to equal the magnitude of $\vec{a}$ plus the magnitude of $\vec{b}$. A little thought leads to the conclusion that vectors $\vec{a}$ and $\vec{b}$ need to have the same direction (this is justified by the Triangle Inequality from Linear Algebra [MATH 2010]; see my online notes on Section 1.2. The Norm and Dot Product and Theorem 1.5). We can take $\boxed{\vec{a}=\hat{\imath}+\hat{\jmath} \text{ and } \vec{b}=3\hat{\imath}+3\hat{\jmath}}$ (or any nonnegative multiple of $\vec{a}$), so that $\vec{c}=\vec{a}+\vec{b}=4\hat{\imath}+4\hat{\jmath}$. The by Equation (3-6), $a=\sqrt{1^2+1^2}=\sqrt{2}$, $\vec{b}=\sqrt{3^2+3^2}=\sqrt{18}=3\sqrt{2}$, and $c=\sqrt{4^2+4^2}=\sqrt{32}=4\sqrt{2}$. Therefore, $a+b=\sqrt{2}+3\sqrt{2}=4\sqrt{2}=c$, as needed.

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**Solution.** **(b)** From the definition of $-\vec{a}$ and Equation (3-4), we conclude that $(\vec{a}+\vec{b})-\vec{a}=(\vec{a}-\vec{b})-\vec{a}$, or (by associativity and commutativity, (3-2) and (3-3)) $(\vec{a}-\vec{a})+\vec{b}=(\vec{a}-\vec{a})-\vec{b}$, or $\vec{0}+\vec{b}=\vec{0}-\vec{b}$, or $\vec{b}=-\vec{b}$, or $\vec{b}=\vec{0}$. Therefore, we take $\boxed{\vec{a} \text{ to be any vector in } \mathbb{R}^2 \text{ and } \vec{b}=\vec{0}}$.

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**(c)** We have defined vector addition such that in $\mathbb{R}^2$, vectors $\vec{a}$, $\vec{b}$, and $\vec{c}=\vec{a}+\vec{b}$ form a triangle with edges of lengths $a$, $b$, and $c$ (see Figure 3-2(b)). The relationship $a^2+b^2=c^2$ in a triangle is only satisfied when the triangle is a right triangle with hypotenuse of length $c$ by the Pythagorean Theorem (or, technically, the converse of the Pythagorean Theorem). So we need $\vec{a}$ and $\vec{b}$ to be perpendicular. We take $\boxed{\vec{a}=\vec{\imath} \text{ and } \vec{b}=\vec{\jmath}}$ so that $\vec{c}=\vec{\imath}+\vec{\jmath}$. Then $a=1$, $b=1$, and $c=\sqrt{1^2+1^2}=\sqrt{2}$, and hence $a^2+b^2=1^2+1^2=2=(\sqrt{2})^2=2=c$, as needed. □

# Question 3.6 (continued)

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# Problem 3-2.12

**Problem 3-1.12.** A car is driven east for a distance of 50 km, then north for 30 km, and then in a direction 30° east of north for 25 km. Sketch the vector diagram and determine **(a)** the magnitude and **(b)** the angle of the car's total displacement from its starting point.

**Solution.** We introduce Cartesian coordinates with East lying along the positive horizontal axis and North lying along the positive vertical axis.

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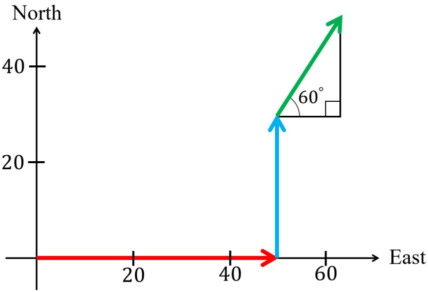

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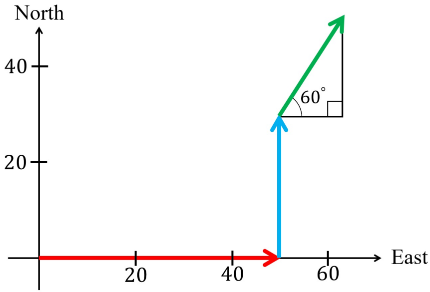
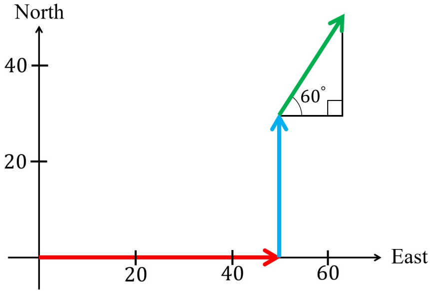


# Problem 3-2.12 (continued 1)

**Solution (continued).**




Let the red eastward vector be $\vec{a}$, the blue northward vector be $\vec{b}$, and the other green vector be $\vec{c}$. Then $\vec{a} = 50\vec{\imath}$ km, and $\vec{b} = 30\vec{\jmath}$ km. We use the drawn right triangle to find the components of $\vec{c}$ (we can also use Equation (3-5)). We have $\cos 30^\circ = c_x/c = c_x/25$ or $c_x = 25\cos 30^\circ = 25/2$ km, and $\sin 30^\circ = c_y/c = c_y/25$ or $c_y = 25\sin 30^\circ = 25\sqrt{3}/2$ km. Hence $\vec{c} = (25/2)\vec{\imath} + (25\sqrt{3}/2)\vec{\jmath}$.

# Problem 3-2.12 (continued 2)

**Problem 3-1.12.** A car is driven east for a distance of 50 km, then north for 30 km, and then in a direction 30° east of north for 25 km. Sketch the vector diagram and determine **(a)** the magnitude and **(b)** the angle of the car's total displacement from its starting point.

**Solution (continued).** The resultant position vector is then $\vec{a} + \vec{b} + \vec{c} = (50 + 25/2)\vec{\imath} + (30 + 25\sqrt{3}/2)\vec{\jmath}$.
**(a)** The magnitude of the resultant position vector is

$$\sqrt{(50 + 25/2)^2 + (30 + 25\sqrt{3}/2)^2} \approx \boxed{81.08 \text{ km}}.$$

**(b)** By Equation (3-6), the angle $\alpha$ of the displacement vector (i.e., the resultant position vector) satisfies $\tan\alpha = \dfrac{(30 + 25\sqrt{3}/2)^2}{(50 + 25/2)^2}$, or (since both components of the displacement vector are positive) $\alpha = \tan^{-1}\left(\frac{30+25\sqrt{3}/2}{50+25/2}\right) \approx \boxed{39.57^\circ}$. That is, the direction is 39.57° North of East (or 50.43° East of North). □

# Problem 3-2.20

**Problem 3-2.20.** An explorer is caught in a whiteout (in which the snowfall is so thick that the ground cannot be distinguished from the sky) while returning to base camp. He was supposed to travel due north for 5.6 km, but when the snow clears, he discovers that he actually traveled 7.8 km at $50^\circ$ north of due east. **(a)** How far and **(b)** in what direction must he now travel to reach base camp?

**Solution.** We represent the displacement vector that the explorer should have used with the red vector given here, the explorer's actual displacement with the blue vector, and the displacement vector that he must travel to reach base camp as the green vector.

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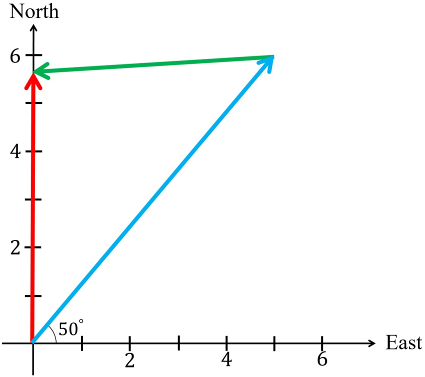

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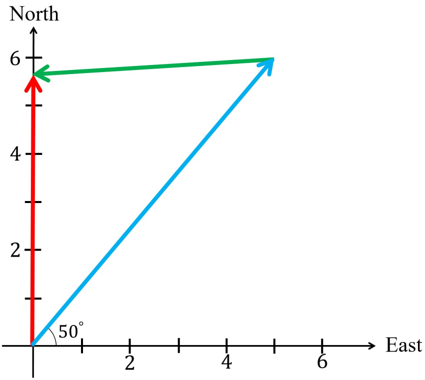
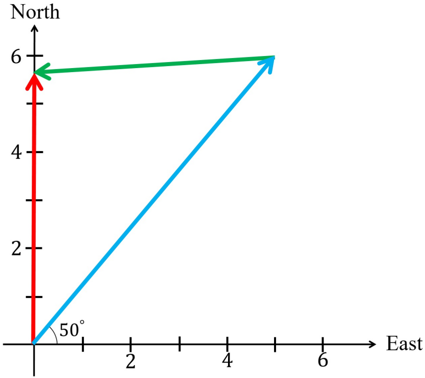


# Problem 3-2.20 (continued 1)

**Solution (continued).**




The red vector is $5.6\vec{\jmath}$ km, the blue vector, by Equation (3.5), is (to two significant figures) $7.8\cos 50\vec{\imath} + 7.8\sin 50\vec{\jmath} \approx 5.0\vec{\imath} + 6.0\vec{\jmath}$ km. Since the red vector equals the blue vector plus the green vector, then the green vector is approximately $(5.6\vec{\jmath}) - (5.0\vec{\imath} + 6.0\vec{\jmath}) = -5.0\vec{\imath} - 0.4\vec{\jmath}$ km.

# Problem 3-2.20 (continued 2)

**Problem 3-2.20.** An explorer is caught in a whiteout (in which the snowfall is so thick that the ground cannot be distinguished from the sky) while returning to base camp. He was supposed to travel due north for 5.6 km, but when the snow clears, he discovers that he actually traveled 7.8 km at $50^\circ$ north of due east. **(a)** How far and **(b)** in what direction must he now travel to reach base camp?

**Solution (continued).** The displacement vector that the explorer must follow (the green vector) is approximately $-5.0\vec{\imath} - 0.4\vec{\jmath}$ km.

**(a)** The distance the explorer must follow is the magnitude of the green vector, $\sqrt{(-5.0)^2 + (-0.4)^2} \approx \boxed{5.0 \text{ km}}$ (to two significant figures).

# Problem 3-2.20 (continued 2)

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**Solution (continued).** The displacement vector that the explorer must follow (the green vector) is approximately $-5.0\vec{\imath} - 0.4\vec{\jmath}$ km.

**(a)** The distance the explorer must follow is the magnitude of the green vector, $\sqrt{(-5.0)^2 + (-0.4)^2} \approx \boxed{5.0 \text{ km}}$ (to two significant figures).

**(b)** The direction, $\theta$, he must follow satisfies $\tan\theta \approx (-0.4)/(-5.0) = 0.080$. Now $\tan^{-1}(0.080) \approx 4.6^\circ$, but since both components of the green vector are negative, we add $180^\circ$ to $4.6^\circ$ to get $\boxed{\theta \approx 184.6^\circ}$ (or $4.6^\circ$ South of West). □

# Problem 3-2.20 (continued 2)

**Problem 3-2.20.** An explorer is caught in a whiteout (in which the snowfall is so thick that the ground cannot be distinguished from the sky) while returning to base camp. He was supposed to travel due north for 5.6 km, but when the snow clears, he discovers that he actually traveled 7.8 km at 50° north of due east. **(a)** How far and **(b)** in what direction must he now travel to reach base camp?

**Solution (continued).** The displacement vector that the explorer must follow (the green vector) is approximately $-5.0\vec{\imath} - 0.4\vec{\jmath}$ km.

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# Problem 3-3.40

**Problem 3-3.40.** Displacement $\vec{d}_1$ is in the $yz$ plane $63.0^\circ$ from the positive direction of the $y$ axis, has a positive $z$ component, and has a magnitude of 4.50 m. Displacement $\vec{d}_2$ is in the $xz$ plane $30.0^\circ$ from the positive direction of the $x$ axis, has a positive $z$ component, and has magnitude 1.40 m. What are **(a)** $\vec{d}_1 \cdot \vec{d}_2$, **(b)** $\vec{d}_1 \times \vec{d}_2$, and **(c)** the angle between $\vec{d}_1$ and $\vec{d}_2$?

**Solution.** Vectors $\vec{d}_1$ and $\vec{d}_2$ as described are:

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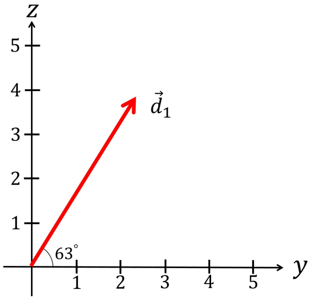


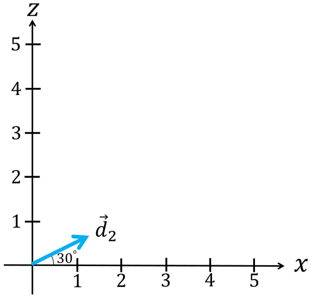

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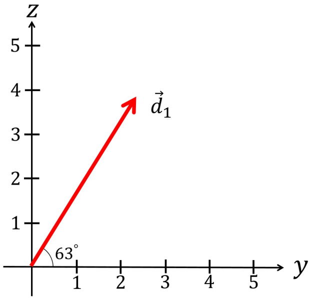


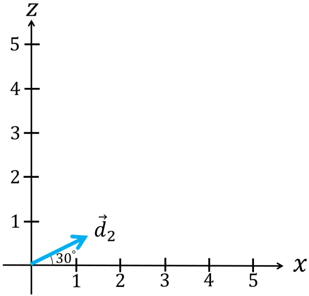

# Problem 3-3.40 (continued 1)

**Problem 3-3.40.** What are **(a)** $\vec{d}_1 \cdot \vec{d}_2$, **(b)** $\vec{d}_1 \times \vec{d}_2$, and **(c)** the angle between $\vec{d}_1$ and $\vec{d}_2$?

**Solution (continued).** By Equation (3-5),
$\vec{d}_1 = (4.50)\cos(63.0°)\vec{\jmath} + (4.50)\sin(63.0°)\vec{k} \approx 0\vec{\imath} + 2.04\vec{\jmath} + 4.01\vec{k}$ m, and
$\vec{d}_2 = (1.40)\cos(30.0°)\vec{\imath} + (1.40)\sin(30.0°)\vec{k} \approx 1.21\vec{\imath} + 0\vec{\jmath} + 0.700\vec{k}$ m.

# Problem 3-3.40 (continued 1)

**Problem 3-3.40.** What are **(a)** $\vec{d}_1 \cdot \vec{d}_2$, **(b)** $\vec{d}_1 \times \vec{d}_2$, and **(c)** the angle between $\vec{d}_1$ and $\vec{d}_2$?

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**(a)** By Equation (3-23)

$$\vec{d}_1 \cdot \vec{d}_2 \approx (0)(1.21) + (2.04)(0) + (4.01)(0.700) = \boxed{2.81 \text{ m}^2}.$$

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**Problem 3-3.40.** What are **(a)** $\vec{d}_1 \cdot \vec{d}_2$, **(b)** $\vec{d}_1 \times \vec{d}_2$, and **(c)** the angle between $\vec{d}_1$ and $\vec{d}_2$?

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$$\vec{d}_1 \cdot \vec{d}_2 \approx (0)(1.21) + (2.04)(0) + (4.01)(0.700) = \boxed{2.81\ \text{m}^2}.$$

**(b)** By Equation (3-27),

$$\vec{d}_1 \times \vec{d}_2 \approx ((2.04)(0.700) - (0)(4.01))\vec{\imath} + ((4.01)(1.21) - (0.700)(0))\vec{\jmath}$$

$$+((0)(0) - (1.21)(2.04))\vec{k} \approx \boxed{1.43\vec{\imath} + 4.85\vec{\jmath} - 2.47\vec{k}\ \text{m}^2}.$$

# Problem 3-3.40 (continued 1)

**Problem 3-3.40.** What are **(a)** $\vec{d}_1 \cdot \vec{d}_2$, **(b)** $\vec{d}_1 \times \vec{d}_2$, and **(c)** the angle between $\vec{d}_1$ and $\vec{d}_2$?

**Solution (continued).** By Equation (3-5),
$\vec{d}_1 = (4.50)\cos(63.0^\circ)\vec{\jmath} + (4.50)\sin(63.0^\circ)\vec{k} \approx 0\vec{\imath} + 2.04\vec{\jmath} + 4.01\vec{k}$ m, and
$\vec{d}_2 = (1.40)\cos(30.0^\circ)\vec{\imath} + (1.40)\sin(30.0^\circ)\vec{k} \approx 1.21\vec{\imath} + 0\vec{\jmath} + 0.700\vec{k}$ m.

**(a)** By Equation (3-23)

$$\vec{d}_1 \cdot \vec{d}_2 \approx (0)(1.21) + (2.04)(0) + (4.01)(0.700) = \boxed{2.81 \text{ m}^2}.$$

**(b)** By Equation (3-27),

$$\vec{d}_1 \times \vec{d}_2 \approx ((2.04)(0.700) - (0)(4.01))\vec{\imath} + ((4.01)(1.21) - (0.700)(0))\vec{\jmath}$$

$$+((0)(0) - (1.21)(2.04))\vec{k} \approx \boxed{1.43\vec{\imath} + 4.85\vec{\jmath} - 2.47\vec{k} \text{ m}^2}.$$

# Problem 3-3.40 (continued 2)

**Problem 3-3.40.** What are **(a)** $\vec{d}_1 \cdot \vec{d}_2$, **(b)** $\vec{d}_1 \times \vec{d}_2$, and **(c)** the angle between $\vec{d}_1$ and $\vec{d}_2$?

**Solution (continued). (c)** Since $\vec{d}_1 \approx 0\vec{\imath} + 2.04\vec{\jmath} + 4.00\vec{k}$ m, and $\vec{d}_2 \approx 1.21\vec{\imath} + 0\vec{\jmath} + 0.700\vec{k}$ m, then the magnitude of $\vec{d}_1$ is $d_1 \approx \sqrt{0^2 + 2.04^2 + 4.01^2} \approx 4.50$ m, and the magnitude of $\vec{d}_2$ is $d_2 \approx \sqrt{1.21^2 + 0^2 + 0.700^2} \approx 1.40$ m. By the definition of scalar product, we have $\vec{d}_1 \cdot \vec{d}_2 = d_1 d_2 \cos\varphi$ where $\varphi$ is the angle between $\vec{d}_1$ and $\vec{d}_2$, so $\cos\varphi = (\vec{d}_1 \cdot \vec{d}_2)/(d_1 d_2)$. We then have (by part (a))

$$\cos\varphi \approx \frac{2.80 \text{ m}^2}{(4.50 \text{ m})(1.40 \text{ m})} = \frac{2.80}{(4.50)(1.40)}.$$

Since $\vec{d}_1$ lies in the first quadrant of the $yz$-plane and $\vec{d}_2$ lies in the first quadrant of the $xz$ plane, then the angle $\varphi$ is between $0^\circ$ and $90^\circ$. Therefore, $\varphi \approx \cos^{-1}\left(\frac{2.80}{(4.50)(1.40)}\right) \approx \boxed{63.5^\circ}$. □

# Problem 3-3.40 (continued 2)

**Problem 3-3.40.** What are **(a)** $\vec{d}_1 \cdot \vec{d}_2$, **(b)** $\vec{d}_1 \times \vec{d}_2$, and **(c)** the angle between $\vec{d}_1$ and $\vec{d}_2$?

**Solution (continued). (c)** Since $\vec{d}_1 \approx 0\vec{\imath} + 2.04\vec{\jmath} + 4.00\vec{k}$ m, and $\vec{d}_2 \approx 1.21\vec{\imath} + 0\vec{\jmath} + 0.700\vec{k}$ m, then the magnitude of $\vec{d}_1$ is $d_1 \approx \sqrt{0^2 + 2.04^2 + 4.01^2} \approx 4.50$ m, and the magnitude of $\vec{d}_2$ is $d_2 \approx \sqrt{1.21^2 + 0^2 + 0.700^2} \approx 1.40$ m. By the definition of scalar product, we have $\vec{d}_1 \cdot \vec{d}_2 = d_1 d_2 \cos\varphi$ where $\varphi$ is the angle between $\vec{d}_1$ and $\vec{d}_2$, so $\cos\varphi = (\vec{d}_1 \cdot \vec{d}_2)/(d_1 d_2)$. We then have (by part (a))

$$\cos\varphi \approx \frac{2.80 \text{ m}^2}{(4.50 \text{ m})(1.40 \text{ m})} = \frac{2.80}{(4.50)(1.40)}.$$

Since $\vec{d}_1$ lies in the first quadrant of the $yz$-plane and $\vec{d}_2$ lies in the first quadrant of the $xz$ plane, then the angle $\varphi$ is between $0^\circ$ and $90^\circ$. Therefore, $\varphi \approx \cos^{-1}\left(\frac{2.80}{(4.50)(1.40)}\right) \approx \boxed{63.5^\circ}$. □