Introduction

Note. This chapter briefly reviews the probability topics one would see in an introductory statistics (course equivalent to ETSU's Foundations of Probability and Statistics-Calculus Based, MATH 2050): the sample space of events, conditional probability, independent events, random variables, expectation, variance, covariance, correlation. Some of these topics might also be covered as part of the junior level ETSU class Applied Combinatorics and Problem Solving (MATH 3340; see Section 1.3. Elementary Probability). Though probability is the basis of statistics, this is not an applied class and will concentrate on the theory of probability. The level of these notes is beyond that of a sophomore level class, but not to the level of a measure theory based probability class (which would require extensive background in graduate-level real analysis and functional analysis). I have online notes for a measure theory based probability course at Measure Theory Based Probability; this is not an official ETSU class either.

0.1. Models

Note. Gut starts his book with: "The object of *probability theory* is to describe and investigate mathematical models of random phenomena, primarily from a theoretical point of view." The field of statistics applies the properties of probability theory to draw conclusions about a population based on properties of a sample from the population. Therefore, in addition to being an area on its own, probability theory

is the foundation of all of statistics.

Note. The focus of probability theory is the "random phenomena." These phenomena are meant to model random experiments that can be repeated (indefinitely), in which the outcomes of the experiments cannot be precisely predicted because of the randomness (as opposed to "deterministic phenomena" which are perfectly predictable, if sufficient knowledge is given of the initial state of the phenomena).

Note. We should make a quick philosophical comment that "deterministic" does not mean "determinable." A system of nonlinear differential equations can have a solution, but due to the nonlinear property there may be no way to exactly find that solution (due to, for example, sensitive dependence on initial conditions). Some more details on these ideas are in my online presentation on A Mathematician Looks at Chaos.

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