0.5. Expectation, Variance, and Moments

Note. In this section, we consider measures of location (the mean and median) and dispersion (the variance) of a random variable.

Definition. Let X be a random variable with distribution function F. The *mean* or *expected value* $E(X)$ is

$$E(X) = \begin{cases} \sum_{k=1}^{\infty} x_k p_X(x_k), & \text{if } X \text{ is discrete,} \\ \int_{-\infty}^{\infty} x f_X(x)\,dx, & \text{if } X \text{ is continuous,} \end{cases}$$

provided the sum or integral is absolutely convergent.

Note. The mean m of a random variable is the value such that $P(X \geq m) \geq 1/2$ and $P(X \leq m) \geq 1/2$.

Definition. The *variance* of random variable X, $\mathrm{Var}(X)$, is

$$\mathrm{Var}(X) = E(X - E(X))^2.$$

Note. The variance of random variable X can be computed as

$$\mathrm{Var}(X) = \begin{cases} \sum_{k=1}^{\infty} (x_k - E(X))^2 p_X(x_k), & \text{if } X \text{ is discrete,} \\ \int_{-\infty}^{\infty} (x - E(X))^2 f_X(x)\,dx, & \text{if } X \text{ is continuous,} \end{cases}$$

provided these series and integrals exist. We can expand the summand and the integrand to get an alternative expression for variance, which in practice is easier to compute: Var $(X) = E(X^2 - (E(X))^2$.

Definition. For random variable X, the *nth moment* is $E(X^n)$ where $n \in \mathbb{N}$. The *nth central moment* is $E(X - E(X))^n$ where $n \in \mathbb{N}$ (provided these exist). The *absolute moments* are $E(|X|^r)$ where $r > 0$. The *absolute central moments* are $E(|X - E(X)|^r)$ where $r > 0$ (provided these exist).

Note. When $n = 1$, the nth moment is the mean. When $n = 2$, nth central moment is the variance. Moments are addressed in Mathematical Statistics 1 (STAT 4047/5047) in Section 1.9. Some Special Expectations.

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