

# Chapter 0. Introduction

## Study Guide

The following is a brief list of topics covered in Chapter 0 of Allan Gut's *An Intermediate Course in Probability*, 2nd Edition (Springer, 2009). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

### Section 0.1. Models.

Random phenomena, deterministic phenomena.

### Section 0.2. The Probability Space.

Relative frequency, stabilization of relative frequencies, intuitive motivation of Kolmogorov's axiomatization of probability theory, finite additivity, sample space, elementary event, events,  $\sigma$ -algebra ( $\sigma$ -field), probability space and the three axioms, probability, countable additivity, biographical information on Andrey Kolmogorov.

### Section 0.3. Independence and Conditional Probabilities.

Independence of two events, pairwise independent events, conditional probability, Law of Total Probability and Bayes' Formula (Theorem 0.3.A), biographical information on Thomas Bayes.

### Section 0.4. Random Variables.

Random variable, integrable functions, The  $\sigma$ -algebra of all events is generated by the intervals of the form  $(-\infty, x]$  (Note 0.4.A), distribution function  $F_X$ , probability function  $P_X$  for a discrete distribution, density function (or probability density function)  $f_X$  of a continuous distribution.

### Section 0.5. Expectation, Variance, and Moments.

Expected value, mean, variance and its computation,  $n$ th moment,  $n$ th central moment, absolute moments, absolute central moments.

### Section 0.6. Joint Distributions and Independence.

Joint distribution function, joint probability function, joint density, independent random variables.

**Section 0.7. Sums of Random Variables, Covariance, Correlation.**

Computation of  $p_{X+Y}$  for independent random variables, convolution formula, computation of  $f_{X+Y}$ , variance of  $aX + bY$ , covariance, correlation coefficient, uncorrelated.

**Section 0.8. Limit Theorems.**

Weak Law of Large Numbers, sample mean, Central Limit Theorem, Markov's Inequality, Chebyshev's Inequality.

**Section 0.9. Stochastic Processes.**

Stochastic process, discrete/continuous stochastic process, Poisson process.

**Section 0.10. The Contents of the Book.**

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