Chapter 4. Basic Concepts of Probability

Section 4.1. Introduction

Note. We start in Chapter 4 with our exploration of measure theory based on probability theory. We assume as background a knowledge of measure theory consistent with that given in Chapter 17 ("General Measure Spaces: Their Properties and Construction") and Chapter 18 ("Integration Over General Measure Spaces") of Royden and Fitzpatrick's *Real Analysis*, 4th edition (Prentice Hall, 2010). In this chapter, measure theory plays a minor role until we get to Section 4.6 ("Random Variables"). We start with a somewhat informal approach.

Note/Definition. When performing a "random experiment," the set Ω of all possible outcomes is the *sample space* for the experiment.

Definition. A *σ-field* on set Ω is a collection of subsets of Ω which includes Ω, is closed under countable unions, countable intersections, and complements.

Note. A "σ-field" is the same as a "σ-algebra." In these probability notes we'll use the term σ-field.

Definition. A measure space $(\Omega, \mathcal{F}, P)$ where $\mathcal{F}$ is a σ-field on sample space Ω, P is a measure defined on $\mathcal{F}$, and $P(\Omega) = 1$ is a *probability space.* An element in $\mathcal{F}$ is an *event.*

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