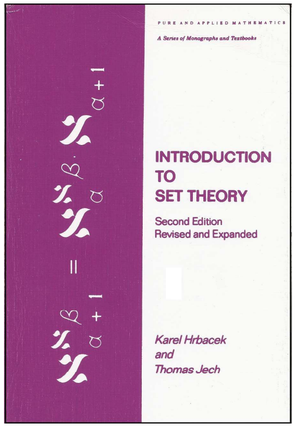
Introduction to Set Theory

Chapter 1. Sets

1.4. Elementary Operations on Sets—Proofs of Theorems



Theorem 1.4.A

Theorem 1.4.A. Let A, B, and C be sets. Then (Distributivity): $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Proof. Let $x \in A \cap (B \cup C)$. Then $x \in A$ and $x \in B \cup C$. Therefore, $x \in A$ and either $x \in B$ or $x \in C$. So either ($x \in A$ and $x \in B$) or ($x \in A$ or $x \in C$); that is, either $x \in A \cap B$ or $x \in A \cap C$. Hence $x \in (A \cap B) \cup (A \cap C)$. Ergo, every element of $A \cap (B \cup C)$ is in $(A \cap B) \cup (A \cap C)$ and so $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$.

Let $x \in (A \cap B) \cup (A \cap C)$. Then either $x \in A \cap B$ or $x \in A \cap C$. So either ($x \in A$ or $x \in B$) or ($x \in A$ and $x \in C$). If $x \in A$ and $x \in B$ then $x \in A$ and $x \in B \cup C$ (since $B \subseteq B \cup C$), and so $x \in A \cap (B \cup C)$. If $x \in A$ and $x \in C$ then $x \in A$ and $x \in B \cup C$ (since $C \subseteq B \cup C$), and so $x \in A \cap (B \cup C)$. In either case of ($x \in A$) and $x \in B$) and ($x \in A$ and $x \in C$) we have $x \in A \cap (B \cup C)$. Ergo $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$. As observed above, these imply that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. □