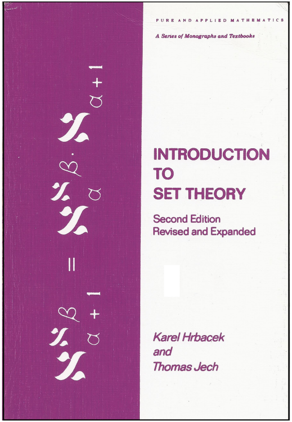
Introduction to Set Theory

Chapter 1. Sets

1.4. Elementary Operations on Sets—Proofs of Theorems



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