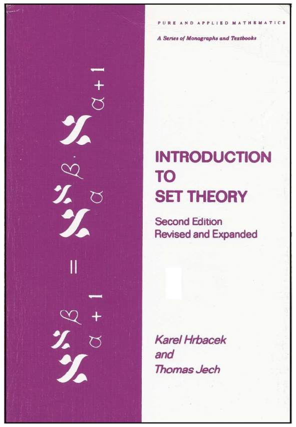
Introduction to Set Theory

Chapter 2. Relations, Functions, and Orderings

2.1. Ordered Pairs—Proofs of Theorems



Theorem 2.1.2

Theorem 2.1.2. $(a, b) = (a', b')$ if and only if $a = a'$ and $b = b'$.

Proof. If $a = a'$ and $b = b'$ then $\{a\} = \{a'\}$ and $\{a, b\} = \{a', b'\}$ so $(a, b) = \{\{a\}, \{a, b\}\} = \{\{a'\}, \{a', b'\}\} = (a', b')$.

Suppose $(a, b) = (a', b')$; that is, suppose $\{a\{a\}, \{a, b\}\} = \{\{a'\}, \{a', b'\}\}$. First, if $a \neq b$ then $\{a\} \neq \{a', b'\}$ (since $b' \in \{a', b'\}$ but $b \notin \{a\}$) and so $\{a\} = \{a'\}$ and $\{a, b\} = \{a', b'\}$. So $a = a'$ and then we must have $b = b'$ (since $b \neq a = a'$). Second, if $a = b$ then $\{\{a\}, \{a, b\}\} = \{\{a\}, \{a, a\}\} = \{\{a\}\}$. So $\{\{a\}\} = \{\{a'\}, \{a', b'\}\}$ and we have $\{a\} = \{a'\}$ and $\{a\} = \{a', b'\}$. That is, $a = a'$ and $a = a' = b'$ or $a = a'$ and $b = b'$, as claimed. □