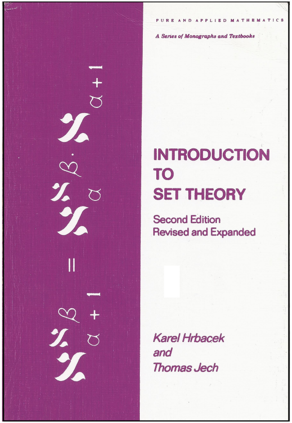
Introduction to Set Theory

Chapter 2. Relations, Functions, and Orderings

2.1. Ordered Pairs—Proofs of Theorems



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Suppose $(a, b) = (a', b')$; that is, suppose $\{a\{a\}, \{a, b\}\} = \{\{a'\}, \{a', b'\}\}$. First, if $a \neq b$ then $\{a\} \neq \{a', b'\}$ (since $b' \in \{a', b'\}$ but $b \notin \{a\}$) and so $\{a\} = \{a'\}$ and $\{a, b\} = \{a', b'\}$. So $a = a'$ and then we must have $b = b'$ (since $b \neq a = a'$).

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