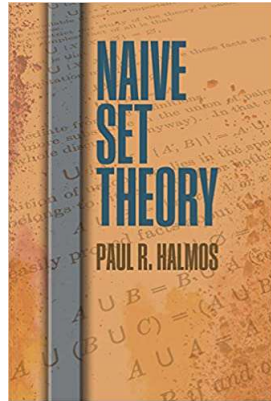


Naive Set Theory

Section 4. Unions and Intersections—Proofs of Theorems



Theorem 4.A

Theorem 4.A. For sets A , B , and C we have

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \text{ and } A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

Proof of Distribution of Union over Intersection. Suppose $x \in A \cup (B \cap C)$. Then either $x \in A$ or $x \in B \cap C$. If $x \in A$, then $x \in A \cup B$ and $x \in A \cup C$, so that $x \in (A \cup B) \cap (A \cup C)$. If $x \in B \cap C$ then $x \in B$ and $x \in C$, so that $x \in A \cup B$ and $x \in A \cup C$, and hence $x \in (A \cup B) \cap (A \cup C)$. That is, if $x \in A \cup (B \cap C)$ then $x \in (A \cup B) \cap (A \cup C)$, or equivalently $A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$.

Next, suppose $x \in (A \cup B) \cap (A \cup C)$. Then $x \in A \cup B$ and $x \in A \cup C$. Notice that if $x \notin A$ then we must have both $x \in B$ and $x \in C$; that is, $x \in B \cap C$ and hence $x \in A \cup (B \cap C)$. If $x \in A$, then $x \in A \cup (B \cap C)$. That is, if $x \in (A \cup B) \cap (A \cup C)$ then $x \in A \cup (B \cap C)$, or equivalently $(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$. Therefore, by the Axiom of Extension, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, as claimed. \square