Chapter 1. Sets

Section 1.1. Introduction to Sets

Note. Informally, "a set is any collection, group, or conglomerate" (see page 1 of the Hrbacek and Jech). The text quotes Georg Cantor, a founder of set theory, as defining:

> "A set is a collection into a whole of definite, distinct objects of our intuition or our thought. The objects are elements (members) of the set" (page 1).

Cantor's definition, as the text emphasizes, implies that sets are not physical objects but instead are (like all mathematical objects) ideas.

Note. We can't define everything, since definition must be in terms of other concepts which, themselves, would be defined in terms of other concepts, ... So we leave the definition of *set* and *element* with a fairly intuitive status.

Note. The text describes Russell's Paradox as follows:

> "Consider for example the 'set' R of all those sets which are not elements of themselves. In other words, R is a set of all sets x such that $x \notin x$ ($\in$ reads 'belongs to,' $\notin$ reads 'does not belong to'). Let us now ask whether $R \in R$. If $R \in R$, then R is not an element of itself (because no element of R belongs to itself), so $R \notin R$; a contradiction.

> Therefore, necessarily $R \notin R$. But then R is a set which is not an element of itself, and all such sets belong to R. We conclude that $R \in R$; again a contradiction" (page 2).

Note. Similarly, the existence of a "set of all sets" leads to contradictions (see Exercises 1.3.3 and 1.3.6). The lesson in this observation and in Russell's Paradox is that simply defining a certain set does not imply its existence. We need a collection of axioms which give the existence of certain sets (such as the empty set) and methods for constructing new sets from known sets. So this course is about *axiomatic set theory*, as opposed to naive set theory which you might encounter at the beginning of a real analysis class; see my online notes from Analysis 1 (MATH 4217/5217) on "Sets and Functions" at: `http://faculty.etsu.edu/gardnerr/4217/notes/1-1.pdf`.

Revised: 1/24/2018