Chapter 2. Relations, Functions, and Orderings

Note. In this chapter we build on the axioms of Chapter 1 and introduce new mathematical structures related to sets.

Section 2.1. Ordered Pairs

Note. In this short section, we define an ordered pair using only set theoretic ideas. In particular, we have not yet defined the natural numbers, so there is no concept of "first" and "second" to use in defining ordered pairs.

Note. Recall that for sets a and b the unordered pair $\{a, b\}$ is known to exist by the Axiom of Pair.

Definition. For given sets a and b, the *ordered pair* is $(a, b) = \{\{a\}, \{a, b\}\}$.

Note. The existence of (z, b) is guaranteed by two applications of the Axiom of Pair. Uniqueness is given in the following theorem.

Theorem 2.1.2. $(a, b) = (a', b')$ if and only if $a = a'$ and $b = b'$.

Definition. For sets a, b, c, d, define an *ordered one tuple* $(a) = a$. Define an *ordered triple* $(a, b, c) = ((a, b), c)$ and an *ordered quadruple* $(a, b, c, d) = ((a, b, c), d)$.

Note. We wait until our definition of the natural numbers to define a general n-tuple, but you can see how this can be done inductively based on the previous definition. Notice that explicitly in terms of sets:

$$(a, b, c) = ((a, b), c) = (\{\{a\}, \{a, b\}, c) = \{\{\{a\}, \{a, b\}\}, \{\{\{a\}, \{a, b\}\}, c\}\}.$$

Note. An alternative approach to defining ordered pairs, triples, and quadruples is to be given in exercise 2.1.6. There, "positions" are indicated using $\varnothing, \{\varnothing\}, \{\{\varnothing\}\}$, and $\{\{\{\varnothing\}\}\}$.

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