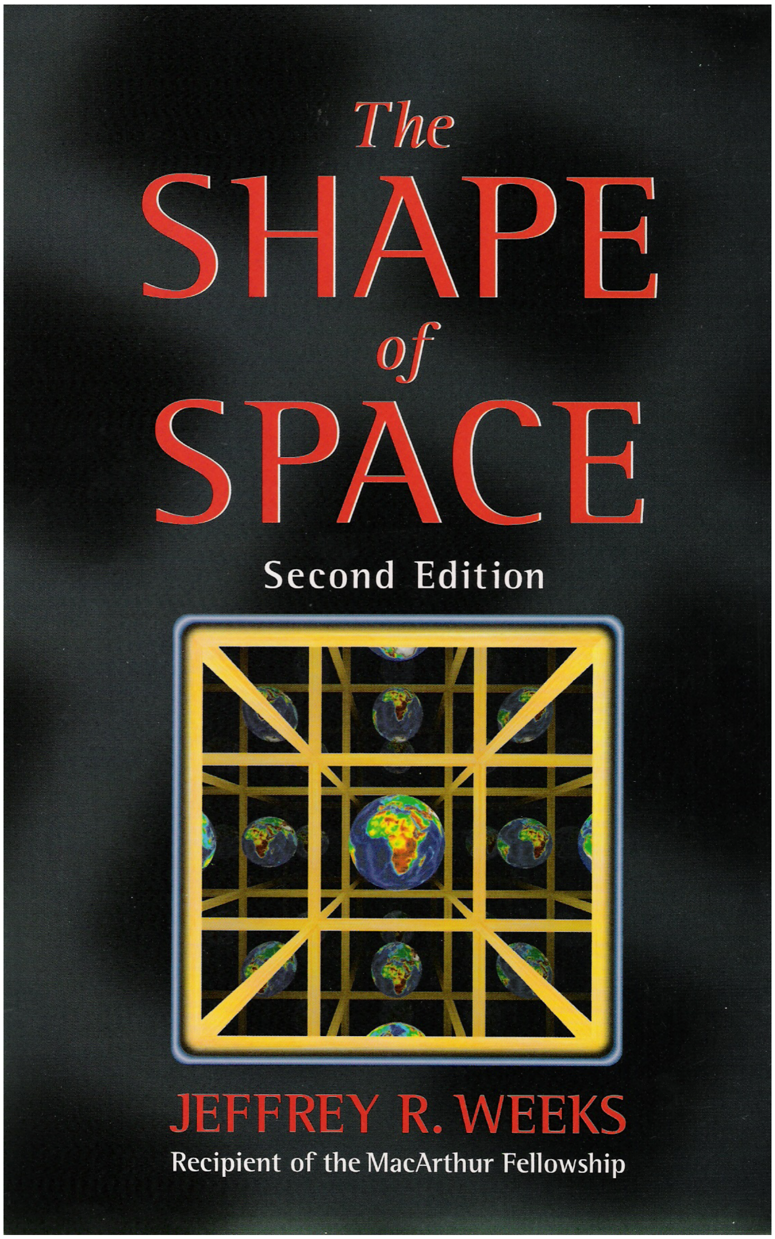
The Global Topology of the Universe

Dr. Bob Gardner

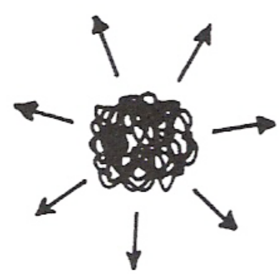
Great Ideas in Science (BIOL 3028)



An Incorrect Picture of the Big Bang

(1) At the moment of the big bang, all matter starts out at a single point in space.

(2) It goes flying off into space in all directions,



(3) and eventually forms galaxies which continue to move further out into space.

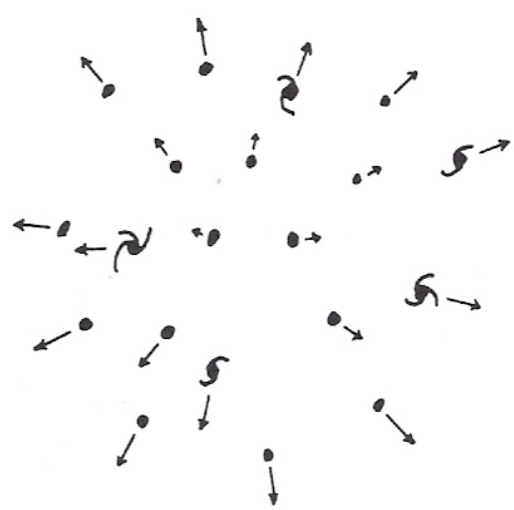
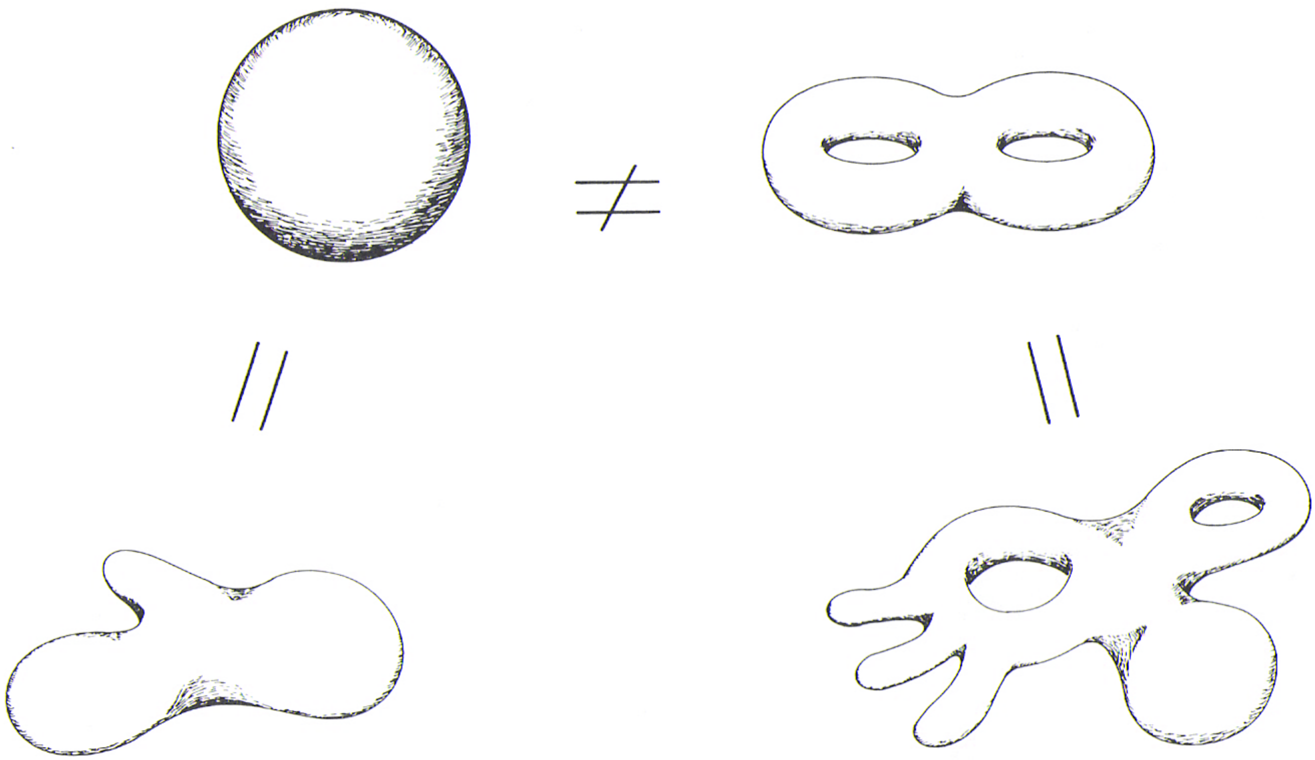


Figure 19.1

Geometry versus Topology

Definition. The aspects of a surface's (or any geometric object's) nature which are unaffected by deformation make up the *topology* of the surface (or object). For example, connectedness and simple-connectedness are topological properties:



Definition. A surface's *geometry* consists of those properties which *do* change when the surface is deformed. For example, curvature, area, distances, and angles are geometric properties.

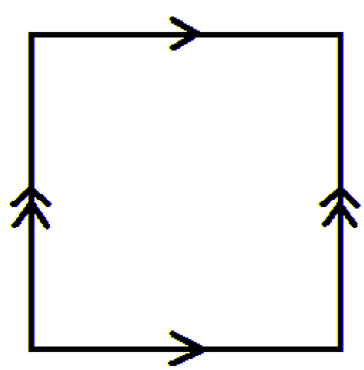
Local versus Global Properties

Definition. *Local properties* of a manifold are those which are observable within a small region of the manifold. *Global properties* require consideration of the manifold as a whole.

Note. A surface is "locally 2-dimensional." We live in a universe that is locally 3-dimensional. That is, our universe is a 3-*manifold*.

Note. When discussing properties of surfaces, we often take the perspective of a 2-dimensional creature living within the surface (a "Flatlander").

Definition. A *flat* 2-*torus* can be visualized as follows:



"Connect" opposite sides according to the arrows.

The "connections" are not physically performed, only conceptually performed (so an insect crawling off the right side appears on the left side.)

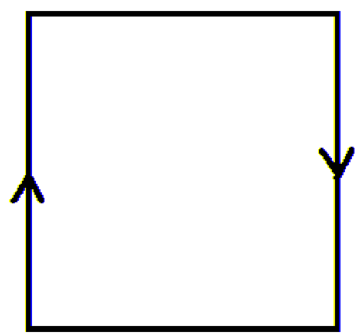
Note. Now let's construct a flat 3-torus. We start with a cube. Conceptually join the front and back of the cube, the left and the right sides of the cube, and the top and the bottom of the cube. In this room, that corresponds to joining the front wall and the back wall, the left wall and the right wall, and the floor and the ceiling. Again, the connections are conceptual. This means that if we throw a ball out the back of the room, it will reappear at the front of the room. This reflects the path that a photon would follow. therfore, if you look straight out the back of the room, you see the back of your own head!

Note. There are only 6 directions in which one can look to see the back of one's own head (well. . .). Since these directions "special" in that sense, a 3-torus is not isotropic. However the "structure" of space is the same at all points, and the space is *homogeneous*.

Note. Another interesting 3-manifold is the 3-sphere. It is best thought of by making analogies with the 2-sphere. Here's the story of what we would see in a 3-sphere. If you are the sole inhabitant of such a space, then you would see yourself in every direction you look. Suppose the circumference of this 3-sphere is C and suppose I am in this space with you. If I start to fly away from you, then you see my image get smaller and smaller until I am a distance of $C/4$ from you. Then my image starts to get larger. When I am only 3 feet from the point antipodal to you (which lies a distance of $C/2$ from you), then I appear the same as I would when I am only 3 feet from you. When I arrive at the antipodal point, you see my image fill the entire sky!

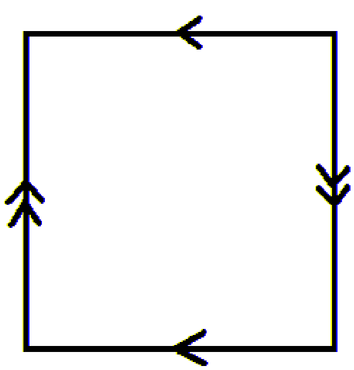
Definition. A path in a manifold that brings a traveler back to his point of origin in a mirror-reversed state is called an *orientation reversing path*. Manifolds with such paths are *nonorientable*. So far, we have only discussed *orientable* manifolds.

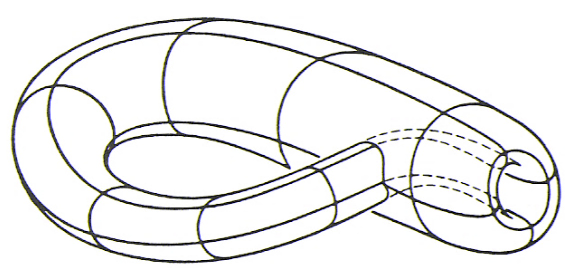
Definition. A *Möbius strip* is constructed by taking a rectangle and joining opposite sides after twisting the rectangle:



This has the curious property that it only has one side and one edge. A Möbius strip is a nonorientable surface.

Note. We can join the top and the bottom of the Möbius strip to generate a *Klein bottle*. This is an example of a nonorientable 2-manifold. Unfortunately, we cannot visualize a Klein bottle in 3 dimensions.





Note. We can mentally construct nonorientable 3-manifolds in a manner similar to the construction of the 3-torus. Consider connecting the floor and ceiling, and the front and back of this room. Now we'll connect the sides of the room by conceptually connecting the front part of the left side to the back part of the right side (this corresponds to the "flip" made in constructing the Möbius strip).

Note. When discussing local and global properties, we are mostly concerned with "local geometry" and "global topology." A flat torus and a doughnut have the same global topology, but different local geometries (consider the sum of angles of a triangle, eg.). On the other hand, a flat torus and a plane have the same local geometry, but different global topology.

Closed versus Open Manifolds

Definition. A manifold is *closed* if it is finite (or more accurately, if it is *bounded*). A manifold is *open* if it is infinite (or *unbounded*).

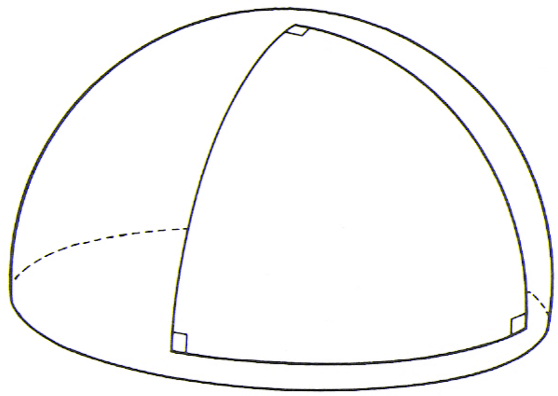
Note. When we use the term "manifold," we refer to objects without boundaries. For example, a circle is a manifold (locally 1-dimensional) but a disk (which is locally 2-dimensional) is not.

Note. There are 6 Euclidean, closed manifolds.

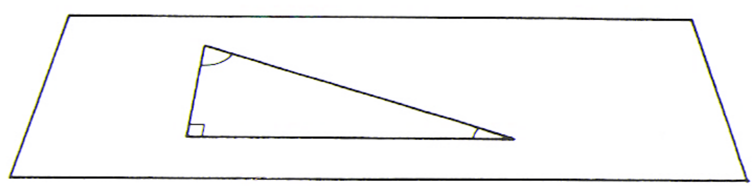
Curvature

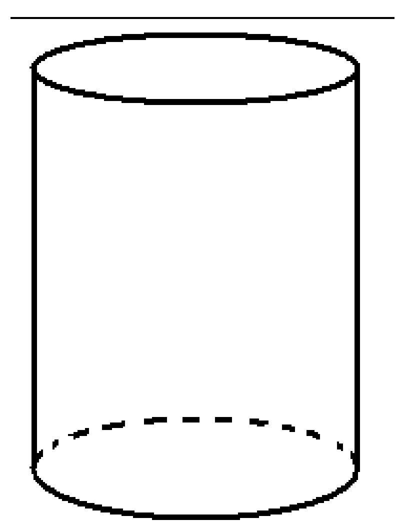
Note. We illustrate curvature very informally with three examples (each a 2-manifold).

1. A sphere has positive curvature:

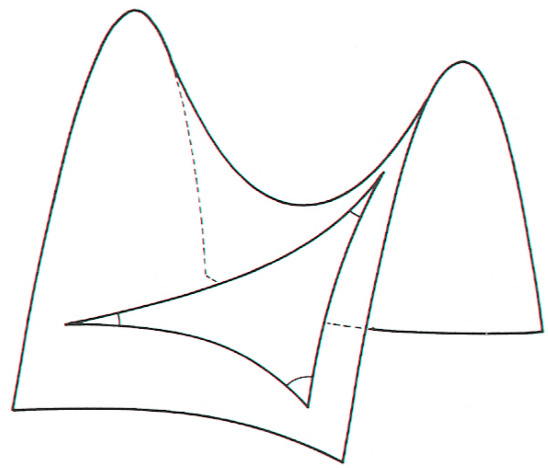


2. A plane and a cylinder have zero curvature:





3. A "saddle" has negative curvature:



Definition. A surface with positive curvature is said to have *elliptic geometry.* A surface with zero curvature is said to have *Euclidean geometry.* A surface with negative curvature is said to have *hyperbolic geometry.* (However, the term "geometry" implies a certain homogeneity of a surface — namely, that the surface must be of constant curvature. The sphere, plane, and cylinder are of constant curvature, but the saddle surface is not.)

Local Geometry of the Universe

Note. As we have seen, the universe may have any of the three above geometries, depending on its density:

Geometry	**Fate**	**Density**
Elliptic	Recollapse	$\rho > 3H/(8\pi G)$
Euclidean	Eternal Expansion (barely)	$\rho = 3H/(8\pi G)$
Hyperbolic	Eternal Expansion	$\rho < 3H/(8\pi G)$

Note. An elliptic universe must have finite volume and therefore deserves the term "closed." However, a common misconception is to describe a Euclidean or hyperbolic universe as necessarily "open' (i.e., infinite). This is NOT THE CASE! The proper terms to use are (geometrically): elliptic, Euclidean, hyperbolic, OR (curvature): positive, zero, negative (respectively). As we shall see, there are possible (even, in a sense, *probable*) Euclidean and hyperbolic geometries on closed 3-manifolds. This leads us to questions of global topology for the 3-manifold which is our universe.

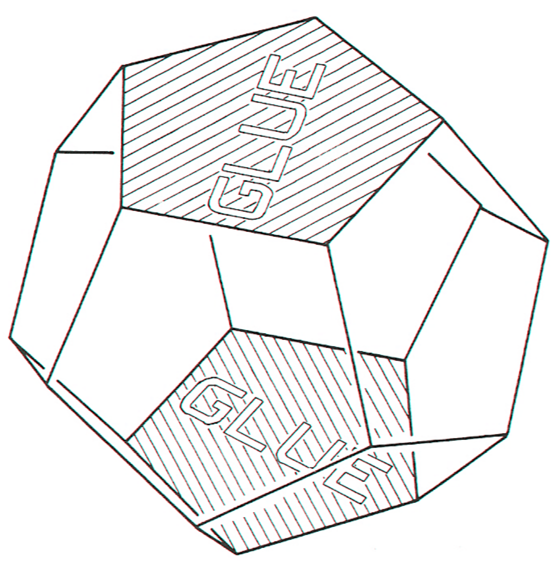
Global Topology of the Universe

Note. We can best visualize certain 3-manifolds by throwing out a dimension and imagining ourselves as flatlanders. An example of a closed 2-manifold with positive curvature is then a sphere (denoted S^2). An example of an open 2-manifold with hyperbolic geometry is the saddle (denoted H^2). Just add a dimension and you have two possible models for the universe (denoted S^3 and H^3, respectively).

Recall. The flat 3-torus is constructed by conceptually joining opposite faces of a cube. This is denoted T^3 and is an example of a closed Euclidean 3-manifold.

Note. S^3, H^3, and $E^3 = \mathbb{R}^3$ are the only homogeneous and isotropic 3-manifolds.

Note. The Seifert-Weber space is constructed by conceptually adhering opposite faces of a dodecahedron, after giving them a 3/10 clockwise turn. This space is an example of a closed hyperbolic 3-manifold.



A Correct Picture of the Big Bang
(illustrated via a two-dimensional universe)

(1) Space itself starts off being very small. All the matter of the universe is crammed into it.

(2) Space expands very rapidly at first.

(3) Eventually the matter is cool enough to begin forming galaxies.

(4) The galaxies continue to move away from each other. The size of each galaxy stays the same.

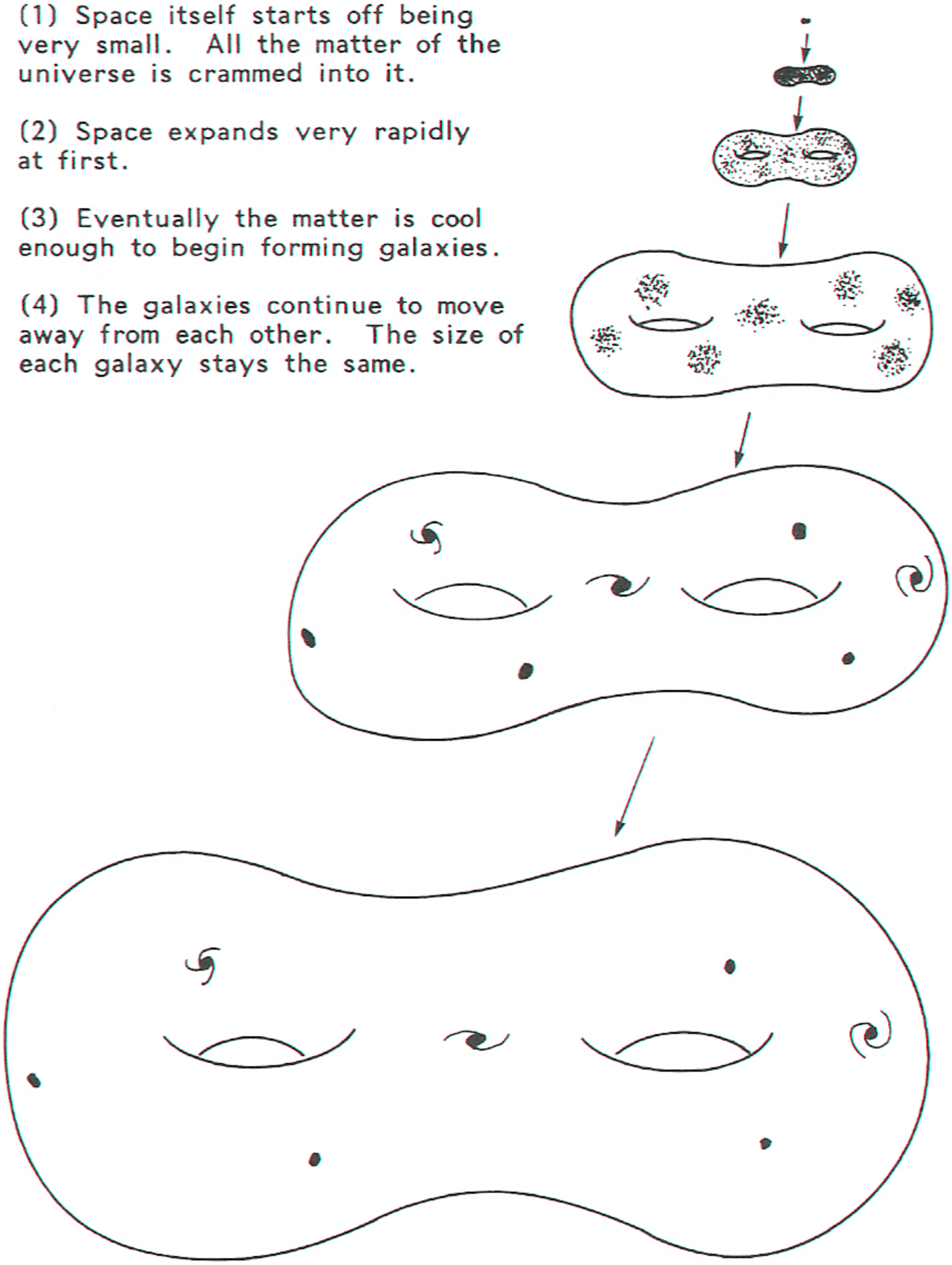


Figure 19.2

Possible Global Topologies of the Universe

	Elliptic	**Euclidean**	**Hyperbolic**
closed	S^3	T^3	Seifert-Weber space
open	NONE	3-D Euclidean Space, E^3	H^3

Note. William Thurston has shown (1970s–80s) that "most" 3-manifolds admit a hyperbolic geometry.

Empirical Evidence

Note. Einstein's general theory of relativity tells us how matter and space interact. This theory only deals with the universe's *local geometry* (and it is the field equations which lead us to the 'fate of the universe" under the different local geometries given above). If we can calculate the universe's density, then we can determine its curvature, local geometry, and ultimate fate. However, this still leaves the question of the universe's global topology.

Note. One could detect that he lives in a closed (i.e., finite) universe by looking for images of himself! That is, we could look for, say, an image of the Virgo cluster appearing far off in the universe. Or, we could look for two images of a given quasar in opposite directions of the sky. However, given the distance scale, these objects simply evolve too quickly for us to recognize them as they were billions of years ago.

Note. Consider the cosmic microwave background (CMB). As we see it, it represents a sphere of radiation (called the "surface of last scattering") with us at the center. In a closed universe, this sphere could intersect itself ad the intersection would result in a circle. It might be possible, then, to see the same circles of intersection in two different areas of the sky. There are tiny fluctuations in the CMB, and these circles could be detected by looking for similar patterns in the fluctuations. With the circles in hand, a precise model of the universe could be EMPIRICALLY constructed which reflects not only the local geometry, but also the global topology.

Note. The Cosmic Background Explorer (COBE) launched around 1990 did not have sufficient resolution to detect these circles (its resolution was only 7° — 14 times the apparent diameter of the moon). However, NASA plans to launch the Microwave Anisotropy Satellite (MAP) in Fall 200 which would have sufficient resolution to detect these circles, In addition, the European Space Agency is planning to launch a satellite called the Planck Explorer in the early 2000s which will have a resolution even better than that of MAP. Therefore, we should see these fundamental questions resolved within the next 7 to 10 years!

References

1. Cornish, N. J., Spergel, D. N. and Starkman, G. D., Measuring the Topology of the Universe, *Proceedings of the National Academy of Science, U.S.A.* **95**, 82–84.

2. Cornish, N. J. and Weeks, R., Measuring the Shape of the Universe, *Notices of the American Mathematical Society* **45**(11), 1463–1471.

3. Cowen, R., Cosmologists in Flatland: Searching for the Missing Energy, *Science News* **153**, 139–141.

4. Peterson, I., Circles in the Sky: Detecting the Shape of the Universe, *Science News* **153**, 123–125.

5. Thurston, W., *Three-Dimensional Geometry and Topology, Volume* 1, 1997, Princeton University Press, Princeton.

6. Weeks, J. R., *The Shape of Space: How to Visualize Surfaces and Three-Dimensional Manifolds*, Monographs and Textbooks in Pure and Applied Mathematics #96, 1985, Marcel Dekker, New York.

7. The Microwave Anisotropy Satellite: `map.gsfc/nasa.gov`

8. The Planck Explorer webpage: `astr.estec.esa.nl/SA-general/Projects/Planck`

9. "The Shape of Space" (an ETSU Observatory open house presentation by Bob Gardner) webpage: `www.etsu.edu/physics/etsuobs/starprty/120598bg/startit.htm`