Special Relativity
Homework Set 1

1. In the Michelson-Morley experiment, $L_1 = L_2 = 11$m, and the light used had a wavelength $\lambda \approx 5.9 \times 10^{-7}$m. The period T of this light, given by $T = \lambda/c$, is approximately 2×10^{-15}sec. Let $N = (\Delta t - \Delta t')/T$, the number of periods contained in the change in time delay.

- **(a)** Assuming the Earth's speed through the ether is 3×10^4m/sec, verify that $N \approx 0.37$ (on the basis of the ether theory), i.e., the interference pattern should shift by 37% of the distance between consecutive bands.
- **(b)** In actuality, Michelson and Morley observed a pattern shift less than 1% of the distance between consecutive bans, i.e., $N \approx 0.01$. Show this implied that the "ether wind," if it exists at all, is "blowing" at less than one-sixth the Earth's orbital speed.

In the following two exercises, assume that any object moving through the ether undergoes a contraction by the factor $(1 - v^2/c^2)^{1/2}$ in the direction of its motion, as suggested by Fitzgerald. The travel times t_1 and t_2 are then $t_1 = \dfrac{2L_1}{c}\left(1 - \dfrac{v^2}{c^2}\right)^{-1/2}$ and $t_2 = \dfrac{2L_2}{c}\left(1 - \dfrac{v^2}{c^2}\right)^{-1/2}$. (There is now no predeicted shift when the apparatus is rotated.)

2. Suppose we assume that the sun is at rest in the ether. Let u ($\approx 3 \times 10^4$m/sec) be the Earth's orbital velocity, and let w (≈ 460m/sec) be the surface velocity of the Earth due to the Earth's rotation on its axis. Compute the interference pattern shift ΔN that would be expected over the course of a day when the speed of the apparatus varies from $v = u + w$ to $v' = u - w$. Hint: We can show that $\Delta N = \dfrac{\Delta L}{\lambda}\left(\dfrac{v^2 - v'^2}{c^2}\right)$.

3. Let us now reject both the ether hypothesis and the contraction hypothesis and assume the speed of light is the same in all directions in all inertial frames. If $\Delta L = L_1 - L_2 \neq 0$, then the light rays from the two arms will arrive at the viewer separated by a time interval of $2\Delta L/c$ seconds. If T is the period of the light used, then the number of periods in this time interval is $N = \dfrac{2\Delta L}{cT}$. In the Kennedy-Thorndike experiment, $\Delta L = 0.16$ m and $T \approx 1.8 \times 10^{-15}$sec.

- **(a)** Compute N.
- **(b)** The equation for N above may be solved for c to obtain $c = \dfrac{2\Delta L}{NT}$. Over six months of observation, N was found to vary less than 0.003. On the assumption that ΔL and T remained constant, show that this implies a variation in the speed of light of about 1.5 m/sec. [Hint: $\Delta c \approx (dc/dN)\Delta N$.]