# Special Relativity
# Homework, Set 3

**1.** The observer in frame $S$ finds that a certain event $A$ occurs at the origin of his coordinate system, and that a second event $B$ occurs $2 \times 10^{-8}$sec later at the point $x = 1200$cm, $y = z = 0$cm. (a) Is there an inertial observer $S'$ for whom these two events are simultaneous? (b) If so, what is the speed and direction of motion of $S'$ relative to $S$?

**2.** We now derive the *relativistic addition of velocities* rule that replaces the classical law invalidated by Einstein's postulates. Here the relative velocity of $S'$ with respect to $S$ will be denoted $\beta_r$. Suppose a missile is fired from rocket $S'$ in a direction parallel to the $x'$ axis with velocity $\beta'$ relative to $S$. If the missile travels $\Delta x'$cm in $\Delta t'$cm of light travel time (as recorded by the rocket clocks), then $\beta' = \Delta x'/\Delta t'$. Similarly, the speed of the missile as measured with laboratory rods and clocks is $\beta = \Delta x/\Delta t$. Using Equatons (91), show that $\beta = \dfrac{\beta' + \beta_r}{1 + \beta_r\beta'}$. (Notice that for $|\beta'| \ll 1$ and $|\beta_r| \ll 1$, the relativistic formula is approximated by the classical rule $\beta = \beta' + \beta_r$.)

**3.** In the previous problem, show that if $\beta' = \pm 1$, then $\beta = \pm 1$ also. (If the missile fired from the rocket is a light photon, then both observers will measure the photon's speed as unity. The invariance of the speed of light [Postulate 2] is therefore embodied in the relativistic law of addition of velocities.)

**4.** Define the imaginary-valued $w$ by $w = it$. Using the identities $\cosh\theta = \cos(i\theta)$ and $\sinh\theta = -i\sin(i\theta)$, show that Equations (89) may be expressed as

$$\begin{aligned} x &= x'\cos(i\theta_r) - w'\sin(i\theta_r) \\ w &= x'\sin(i\theta_r) + w'\cos(i\theta_r) \end{aligned}$$

(Since these equations resemble those for a rotation of coordinates in the Cartesian plane, namely,

$$\begin{aligned} x &= x'\cos(\theta) - y'\sin(\theta) \\ y &= x'\sin(\theta) + y'\cos(\theta), \end{aligned}$$

the Lorentz transformation is sometimes described as an 'imaginary rotation.")