Special Relativity
(Differential Geometry)
Homework, Set 9

1. Show that the geodesic curvature $k_g = \vec{U} \cdot \vec{\alpha}' \times \vec{\alpha}''$ is given by

$$k_g = k\,\vec{U} \cdot \vec{B} = k \cos\theta$$

where k is the curvature of $\vec{\alpha}$ (as a curve in E^3), $\vec{B}$ is the binormal vector of $\vec{\alpha}$, and θ (or $\pi - \theta$) is the angle between the osculating plane of $\vec{\alpha}$ and the tangent plane of the surface.

2. Use the previous exercise to find the geodesic curvature of a circle of latitude on a sphere (a u-parameter curve as we have dealt with it).

3. Prove that a curve on a surface in E^3 is a geodesic of that surface if and only if $k = |k_n|$ at every point of the curve.