Evolution Module

Dr. Bob's Study Guide

6.1. Hardy-Weinberg Equilibrium

TOPICS: Additive and multiplicative Rules for Probabilities, derivation and assumptions of Hardy-Weinberg, calculations of allele frequencies and genotype frequencies under Hardy-Weinberg, Chi-squared test statistic and its use to test for Hardy-Weinberg equilibrium.

6.2. Selection

TOPICS: Fitness values and their meaning, average fitness, change in frequency (Δp), equilibrium values of p, heterozygote advantage model, stable and unstable equilibria and their analysis.

6.5. Inbreeding

TOPICS: Graph theory definitions, construction of a graph from a genealogy, alleles "identical by descent," computation of inbreeding coefficients.

Some Examples

Hardy-Weinberg Example 1. Consider a sample from a population with the following numbers of the three possible genotypes in a one locus/two alleles setting: The number of AA individuals in the sample is 50, the number of Aa alleles in the sample is 50, and the number of aa individuals in the sample is 50. Use a χ^2 test statistic to see if the population is in Hardy- Weinberg equilibrium. Test at 95% confidence, and $\chi^2_{0.050} = 3.84146$ (with 1 degree of freedom).

Solution. We compute an estimate for p of $\hat{p} = (2\times 50+50)(2\times 50+2\times 50+2\times 50) = 150/300 = 1/2$, and so $\hat{q} = 1/2$. The total population size it 150 so, under the assumption of Hardy-Weinberg equilibrium, we get the expected number of AA individuals as $150\hat{p}^2 = 150(1/2)^2 = 37.5$, the expected number of Aa individuals as $150 \times 2\hat{p}\hat{q} = 150 \times 2(1/2)(1/2) = 75$, and the expected number of aa individuals as $150 \times \hat{q}^2 = 150(1/2)^2 = 37.5$. So we have:

Genotype	AA	Aa	aa
O_i	50	50	50
E_i	37.5	75	37.5

This data leads us to the test statistic

$$\sum_{i=1}^{3} \frac{(O_i - E_i)^2}{E_i} = \frac{(50-37.5)^2}{37.5} + \frac{(50-75)^2}{75} + \frac{50-37.5)^2}{37.5} = 16.6667.$$

Since this test value is greater than $\chi^2_{0.050}$, we reject the null hypothesis that the population is in Hardy-Weinberg equilibrium (reject rather strongly!).

Selection Example 1. Find the equilibria of p in the one locus/two alleles fitness model for the lethal recessive case.

Solution. We had for our one locus/two alleles model that

$$\Delta p = \frac{pq}{\overline{w}}(p(w_{11} - w_{12}) + q(w_{12} - w_{22}).$$

In the lethal recessive case, $w_{11} = w_{12} = 1$ and $w_{22} = 0$. Then $\overline{w} = p^2 + 2pq$ and

$$\Delta p = \frac{pq}{p^2 + 2pq}(q) = \frac{(1-p)^2}{2-p}, \text{ if } p \neq 0.$$

So the equilibria are $p = 0$ and $p = 1$. Notice that $\Delta p > 0$ for any $p \in (0, 1)$, and so p is increasing for any $p \in (0, 1)$. So $p = 0$ is an unstable equilibrium and $p = 1$ is a stable equilibrium (notice that since p is an allele frequency, we do not consider the behavior of p for $p < 0$ nor for $p > 1$). Since $\Delta p > 0$, any initial value of p (other than 0) will lead the system to $p = 1$ — that is, to fixation of allele A and extinction of allele a, which is expected since aa is a lethal genotype.

Selection Example 2. Find the equilibria and classify them as to their stability for: $\Delta x = x^2(x-2)(5-x)$.

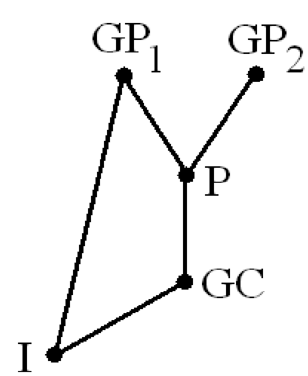
Solution. The equilibria occur where $\Delta x = 0$, so the equilibria are $x = 0$, $x = 2$, and $x = 5$. Consider:

	$(-\infty, 0)$	$(0, 2)$	$(2, 5)$	$(5, \infty)$
test value k	-1	1	3	6
Δx at $x = k$	-18	-4	18	-144
sign of Δx	$-$	$-$	$+$	$-$
behavior of x	DEC $\longleftarrow$	DEC $\longleftarrow$	INC $\longrightarrow$	DEC $\longleftarrow$

Therefore $x = 0$ is a semistable equilibrium, $x = 2$ is an unstable equilibrium, and $x = 5$ is a stable equilibrium.

Inbreeding Example 1. Find the coefficient of inbreeding for a grandparent-grandchild offspring.

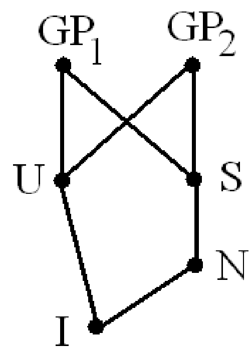
Solution. The genealogy for this mating produces the following graph:



The only common ancestor (relevant to the inbreeding coefficient) of GP_1 and GC is GP_1. We need to find all paths from GP_1 to GC. There is only one: $GP_1 - P - GC$. So

$$F_I = \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

Inbreeding Example 2. Find the coefficient of inbreeding for an uncle-niece offspring.
Solution. The genealogy for this mating produces the following graph:



The common ancestors (relevant to the inbreeding coefficient) of U and N are GP_1 and GP_2. We need to find all paths from U to N. There are 2 such paths: $U - GP_1 - S - N$ and $U - GP_2 - S - N$. So

$$F_I = \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4 = \frac{1}{8}.$$