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Gardner, Robert B. [Gardner, Robert Bentley] (1-ETNS);
Govil, N. K. (1-ABRN)
On the location of the zeros of a polynomial. (English summary)
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The authors generalize the classical Eneström-Kakeya theorem on the location of all zeros of a polynomial $p(z)=\sum a_\nu z^\nu$ of degree n in the closed unit disk, provided its coefficients are positive and form an increasing finite sequence. Theorem: Let $p(z)=\sum_{\nu=0}^n a_\nu z^\nu$ be a polynomial of degree n. If $\operatorname{Re} a_j=\alpha_j$, $\operatorname{Im} a_j=\beta_j$, $j=0,1,2,\cdots,n$, and for some nonnegative integers k, r, and $t\geq 0$, $\alpha_0\leq t\alpha_1\leq t^2\alpha_2\leq\cdots\leq t^k\alpha_k\geq t^{k+1}\alpha_{k+1}\geq\cdots\geq t^n\alpha_n$, and $\beta_0\leq t\beta_1\leq t^2\beta_2\leq\cdots\leq t^r\beta_r\geq t^{r+1}\beta_{r+1}\geq\cdots\geq t^n\beta_n$, then $p(z)$ has all its zeros in the annulus $R_1\leq|z|\leq R_2$, where

$$R_1=\min\left\{\frac{t|a_0|}{2(t^k\alpha_k+t^r\beta_r)-(\alpha_0+\beta_0)-t^n(\alpha_n+\beta_n-|a_n|)},t\right\},$$

$$\begin{aligned}R_2=\max\Bigg\{|a_0|t^{n+1}-t^{n-1}(\alpha_0+\beta_0)-t(\alpha_n+B_n)\\ +(t^2+1)(t^{n-k-1}\alpha_k+t^{n-r-1}\beta_r)\\ +(t^2-1)\Big(\sum_{j=1}^{k-1}t^{n-j-1}\alpha_j+\sum_{j=1}^{r-1}t^{n-j-1}\beta_j\Big)\\ +(1-t^2)\Big(\sum_{j=k+1}^{n-1}t^{n-j-1}\alpha_j+\sum_{j=r+1}^{n-1}t^{n-j-1}\beta_j\Big)\Big/|a_n|,\frac{1}{t}\Bigg\}.\end{aligned}$$

The authors state four corollaries corresponding to the particular cases when the finite sequences α_j and β_j are monotonic and of maximal length $n+1$. *Z. Rubinstein*