MR1350192 (96m:26018) 26D05 26D10 41A17
Gardner, Robert B. [**Gardner, Robert Bentley**] (1-ETNS);
Govil, Narendra K. (1-ABRN)
An L^p inequality for a polynomial and its derivative. (English summary)
J. Math. Anal. Appl. **194** (1995), *no. 3,* 720–726.

Summary: "Let $P(z)=a_n\prod_{\nu=1}^n(z-z_\nu)$, $a_n\neq 0$, be a polynomial of degree n. It is known that if $|z_\nu|\geq K_\nu\geq 1$, $1\leq\nu\leq n$, then for $p\geq 1$, $(\int_0^{2\pi}|P'(e^{i\theta})|^p d\theta)^{1/p}\leq nF_p(\int_0^{2\pi}|P(e^{i\theta})|^p d\theta)^{1/p}$, where $F_p=\{2\pi/\int_0^{2\pi}|t_0+e^{i\theta}|^p d\theta\}^{1/p}$, and $t_0=1+n/\sum_{\nu=1}^n 1/(K_\nu-1)$ if $K_\nu>1$ for all ν, $1\leq\nu\leq n$, $t_0=1$ if $K_\nu=1$ for some ν, $1\leq\nu\leq n$. This inequality is best possible in the case $K_\nu=1$, $1\leq\nu\leq n$, and equality holds for the polynomial $(z+1)^n$. In this paper, we extend the above inequality to values of $p\in[0,1)$ and thus conclude that this inequality in fact holds for all $p\geq 0$."

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