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Decompositions of uniform complete directed multigraphs into each of the orientations of a 4-cycle. (English summary)
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Let D_v^λ denote the uniform complete directed multigraph on v vertices with multiplicity λ. For g a directed graph, a g-decomposition of D_v^λ is a decomposition of the arc multiset of D_v^λ into isomorphic copies of g. There are four orientations of a 4-cycle (a,b,c,d): the 4-circuit and the digraphs X, Y and Z with arc sets $\{(a,b),(b,c),(c,d),(a,d)\}$, $\{(a,b),(b,c),(d,c),(a,d)\}$ and $\{(a,b),(c,b),(c,d),(a,d)\}$, respectively.

With $\lambda=1$, J. Schönheim obtained a necessary and sufficient condition for the existence of a 4-circuit decomposition of D_v^1 [Discrete Math. **11** (1975), 67–70; MR0363996], and the spectrum for the existence of an X- (Y-, Z-) decomposition of D_v^1 was obtained by F. Harary, W. D. Wallis and K. Heinrich [in *Combinatorial mathematics (Proc. Internat. Conf. Combinatorial Theory, Australian Nat. Univ., Canberra, 1977)*, 165–173, Lecture Notes in Math., 686, Springer, Berlin, 1978; MR0526742].

With an arbitrary λ, the authors establish necessary and sufficient conditions for the existence of the decomposition of D_v^λ into each of the orientations of a 4-cycle: a 4-circuit decomposition of D_v^λ exists if and only if $\lambda v(v-1)\equiv 0 \pmod 4$, except $v=4$ and λ odd; an X-decomposition of D_v^λ exists if and only if $\lambda v(v-1)\equiv 0 \pmod 4$, except $v=5$ and $\lambda=1$; a Y-decomposition of D_v^λ exists if and only if $\lambda v(v-1)\equiv 0 \pmod 4$, except $(v,\lambda)=(4,\mathrm{odd}),(5,1)$; and a Z-decomposition of D_v^λ exists if and only if $\lambda v(v-1)\equiv 0$ (mod 4), except $v=0$ and λ odd. The authors employ direct methods of construction, and their constructions lead to necessary and sufficient conditions for the existence of such decompositions which admit cyclic or rotational automorphisms. *Chung Je Cho*