MR2076890 (2005e:30007) 30C10 26C05 26D05
Gardner, Robert B. [Gardner, Robert Bentley] (1-ETNS); **Govil, N. K.** (1-ABRN); **Weems, Amy** (1-ETNS)

Some results concerning rate of growth of polynomials. (English summary)

East J. Approx. **10** (2004), *no. 3,* 301–312.

The main result proved here is the following: if $p(z) = a_0 + \sum_{\nu=t}^{n} a_\nu z^\nu$, $t \geq 1$, is a polynomial of degree $n \geq 2$, $p(z) \neq 0$ for $|z| < K$, $K \geq 1$, and if $M(p, R) = \max_{|z|=R} |p(z)|$, $m = \min_{|z|=K} |p(z)|$, then for $R \geq 1$,

$$M(p,R) \leq \left(\frac{R^n + s_0}{1 + s_0}\right) \|p\| - \left(\frac{R^n - 1}{1 + s_0}\right) m - \left(\frac{R^n - 1}{n} - \frac{R^{n-2} - 1}{n-2}\right) |p'(0)|$$

if $n > 2$, and

$$M(p,R) \leq \left(\frac{R^n + s_0}{1 + s_0}\right) \|p\| - \left(\frac{R^n - 1}{1 + s_0}\right) m - \frac{(R-1)^n}{2} |p'(0)|,$$

if $n = 2$. Here

$$s_0 = K^{t+1} \frac{\left(\frac{t}{n}\right) \frac{|a_t|}{|a_0| - m} K^{t-1} + 1}{\left(\frac{t}{n}\right) \frac{|a_t|}{|a_0| - m} K^{t+1} + 1}.$$

These inequalities sharpen a similar previous result by N. K. Govil [J. Inequal. Appl. **7** (2002), no. 5, 623–631; MR1931257]. *Vania D. Mascioni*

References

1. L. Ahlfors, *Complex Analysis*, McGraw-Hill, New York, 1979. MR0510197
2. N. C. Ankeny and T. J. Rivlin, On a theorem of S. Bernstein, *Pacific J. Math.* **5** (1955), 849–852. MR0076020
3. A. Aziz and M. Dawood, Inequalities for a polynomial and its derivative, *J. Approx. Theory* **53** (1988), 155–162. MR0960052
4. T. N. Chan and M. A. Malik, On Erdös-Lax Theorem, *Proc. Indian Acad. Sci.* **92** (1983), 191–193. MR0767620
5. C. Frappier, Q. I. Rahman and St. Ruscheweyh, New inequalities for polynomials, *Trans. Amer. Math. Soc.* **288** (1985), 69–99. MR0773048
6. N. K. Govil, Some inequalities for deriatives of polynomials, *J. Approx. Theory* **66** (1991), 29–35. MR1113575
7. N. K. Govil, On growth of polynomials, *J. of Inequal. & Appl.* **7** (2002), 623–631. MR1931257
8. N. K. Govil and Q. I. Rahman, Functions of exponential type not vanishing in a half-plane and related polynomials, *Trans. Amer. Math. Soc.*, **137** (1969), 501–517. MR0236385
9. E. Laguerre, *Oeuvres*, Vol. **1**, Gauthier-Villars, Paris, 1898.
10. P. D. Lax, Proof of a conjecture of P. Erdös, *Bull. Amer. Math. Soc.* **50** (1944), 509–513. MR0010731
11. M. A. Malik, On the derivative of a polynomial, *J. London Math. Soc.* **1** (1969), 57–60. MR0249583
12. M. A. Qazi, On the maximum modulus of polynomials, *Proc. Amer. Math. Soc.*

115 (1992), 337–343. MR1113648

13. Q. I. Rahman and J. Stankiewicz, Differential inequalities and local valency, *Pacific J. Math.* **54** (1974), 165–181. MR0374393
14. M. Riesz, Über einen Satz des Herrn Serge Bernstein, *Acta Math.* **40** (1916), 337–347. MR1555142
15. C. Visser, A simple proof of certain inequalities concerning polynomials, *Nederl. Akad. Wetensch. Proc.* **47** (1945), 276–281; *Indag. Math.*, **7** (1945), 81–86. MR0015568

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.