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The Eneström-Kakeya theorem for polynomials of a quaternionic variable. (English summary)

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In this paper, the Eneström-Kakeya theorem is extended to polynomials of a quaternionic variable. It is shown that a quaternionic polynomial with real, nonnegative, monotone increasing coefficients has all its zeros in the unit sphere in the quaternions. Furthermore, using results from the theory of slice regular quaternionic functions, other results for zeros of quaternionic polynomials are proved by dropping the condition of nonnegative coefficients and imposing, for instance, monotone increasing real parts and imaginary parts. The quaternionic setting is different from the complex case since quaternionic polynomials can have an infinite number of zeros consisting of isolated points or 2-spheres. The Maximum Modulus theorem for regular functions introduced by Gentili and Struppa plays a special role in the proof of the results obtained.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.