Decompositions of the Complete Digraph into each of the Orientations of a 4-Cycle which admit a Certain Automorphism

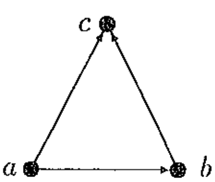
Robert B. Gardner [1], Pam Torbett, Amy Weems
Department of Mathematics
East Tennessee State University
Johnson City, Tennessee 37614 – 0663

Abstract. A decomposition of the complete digraph on v vertices, D_v, is said to be f–*cyclic* if it admits an automorphism consisting of f fixed points and a single cycle of length $v - f$. Necessary and sufficient conditions are given for the existence of f–cyclic decompositions of the complete digraph into each of the four orientations of a 4–cycle.

1 Introduction

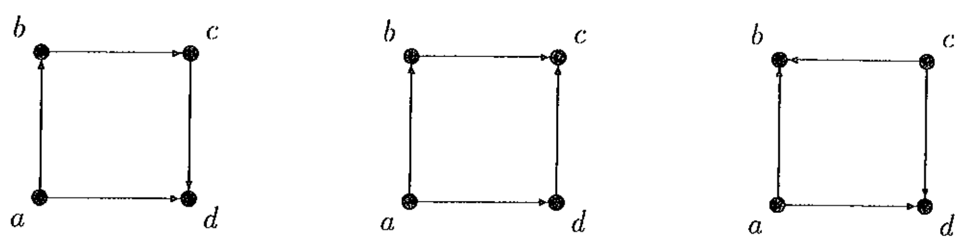
Let D_v denote the complete digraph on v vertices. If g is a digraph, then a g–*decomposition* of D_v is a set $\gamma = \{g_1, g_2, \ldots, g_n\}$ of arc-disjoint subgraphs of D_v such that each g_i (which is called a *block* of the decomposition) is isomorphic to g and $\bigcup_{i=1}^{n} A(g_i) = A(D_v)$, where $A(G)$ is the arc set of digraph G. An *automorphism* of a g–decomposition of D_v is a permutation of the vertex set of D_v which fixes the set γ.

There are two orientations of the 3-cycle: the 3-circuit and the following digraph (called a "transitive triple"):



A decomposition of D_v into 3-circuits is equivalent to a Mendelsohn triple system of order v denoted $MTS(v)$ [9]. A decomposition of D_v into transitive triples is equivalent to a directed triple system of order v, denoted $DTS(v)$ [8].

There are four orientations of the 4-cycle: the 4-circuit and the following:



X Y Z

[1]Research supported by ETSU under RDC grant #96-029/mjr

We represent X as $[a,b,c,d]_X$, Y as $[a,b,c,d]_Y$, and Z as $[a,b,c,d]_Z$. We represent the 4-circuit with arc set $\{(a,b),(b,c),(c,d),(d,a)\}$ by any cyclic shift of $[a,b,c,d]_C$. A 4$-$circuit decomposition of D_v exists if and only if $v \equiv 0$ or 1 (mod 4), $v \neq 4$ [14]. An $X-$decomposition of D_v exists if and only if $v \equiv 0$ or 1 (mod 4), $v \neq 5$; a $Y-$decomposition of D_v exists if and only if $v \equiv 0$ or 1 (mod 4), $v \notin \{4,5\}$; and a $Z-$decomposition of D_v exists if and only if $v \equiv 1$ (mod 4) [6].

A digraph decomposition admitting an automorphism consisting of a single cycle is said to be *cyclic*. A cyclic $MTS(v)$ exists if and only if $v \equiv 1$ or 3 (mod 6), $v \neq 9$ [4] and a cyclic $DTS(v)$ exists if and only if $v \equiv$ 1, 4, or 7 (mod 12) [5]. A cyclic 4-circuit decomposition of D_v exists if and only if $v \equiv 1$ (mod 4) [11]; a cyclic $X-$decomposition of D_v exists if and only if $v \equiv 1$ (mod 4), $v \neq 5$; a cyclic $Y-$decomposition of D_v exists if and only if $v \equiv 1$ (mod 4), $v \neq 5$; and a cyclic $Z-$decomposition of D_v exists if and only if $v \equiv 1$ (mod 4) [3,10].

A decomposition of D_v admitting an automorphism consisting of a fixed point and a cycle of length $v-1$ is said to be *rotational*. A rotational $MTS(v)$ exists if and only if $v \equiv 1, 3$, or 4 (mod 6), $v \neq 10$ [1], and a rotational $DTS(v)$ exists if and only if $v \equiv 0$ (mod 3) [2]. A rotational 4-circuit decomposition of D_v exists if and only if $v \equiv 1$ (mod 4) [13]; a rotational $X-$decomposition of D_v exists if and only if $v \equiv 0$ (mod 4); a rotational $Y-$decomposition of D_v exists if and only if $v \equiv 0$ (mod 4), $v \neq 4$; and a rotational $Z-$decomposition of D_v does not exist [3].

A decomposition of D_v which admits an automorphism consisting of f fixed points, $f > 1$, and a single cycle of length $v - f$ is said to be *$f-$cyclic*. Necessary and sufficient conditions for the existence of a $f-$cyclic $MTS(v)$ are given in [7] and for a $f-$cyclic $DTS(v)$ are given in [12]. The purpose of this paper is to give necessary and sufficient conditions for the existence of a $f-$cyclic $g-$decomposition of D_v where g is an orientation of the 4$-$cycle.

2 The Constructions

In this section we give necessary and sufficient conditions for the existence of a $g-$decomposition of D_v, where g is an orientation of the 4$-$cycle, which admits an automorphism consisting of f fixed points and a cycle of length $v-f$. Throughout this section we suppose the vertex set of D_v is $\{0_0, 1_0, \dots, (f-1)_0, 0_1, 1_1, \dots, (v-f-1)_1\}$ and let the relevant automorphism be $(0_0)(1_0)\dots((f-1)_0)(0_1, 1_1, \dots, (v-f-1)_1)$.

We need a preliminary result before presenting the constructions.

Lemma 2.1 *If π is an automorphism of a $g-$decomposition of D_v, then the fixed points of π form a sub$-g-$decomposition. That is, if $\pi(x_0) = x_0$ and $\pi(y_0) = y_0$ for $(x_0, y_0) \in A(g_0)$, then $\pi(g_0) = g_0$.*

Proof. If $(x_0, y_0) \in A(g_0)$ then by the definition of automorphism, $(\pi(x_0), \pi(y_0)) \in A(\pi(g_0))$. But then $(x_0, y_0) \in A(\pi(g_0))$ and since (x_0, y_0) is in the arc set of exactly one g_i, it must be that $g_0 = \pi(g_0)$. ∎

We have a necessary condition for the existence of a $f-$cyclic 4$-$circuit decomposition of D_v:

Lemma 2.2 *If $v \equiv 0 \ (mod\ 4)$ and $v = f+4$, then a $f-$cyclic $4-$circuit decomposition of D_v does not exist.*

Proof. Suppose such a system does exist. From Lemma 2.1, it follows that arcs of type (a_1, b_1) must be contained in blocks of the form $[w_0, x_1, y_1, z_1]_C$ or $[w_1, x_1, y_1, z_1]_C$. Now each set

$$\{\pi^n([w_0, x_1, y_1, z_1]_C) \mid n \in \mathbf{Z}, w \in \mathbf{Z}_f, \{x, y, z\} \subset \mathbf{Z}_{v-f}\}$$

is of cardinality 4 and so the total number of arcs of type (a_1, b_1) in blocks of the form $[w_0, x_1, y_1, z_1]_C$ is a multiple of 8. Therefore, such a system can have at most one fixed point in blocks of this form, since under our hypotheses D_v contains only 8 arcs of type (a_1, b_1). Therefore each remaining fixed point must be contained in some block of the form $[w_0, x_1, y_0, z_1]_C$ (since each arc of the form (a_0, b_1) is contained in some block). However, such blocks contain two distinct fixed vertices. Therefore, the cardinality of the set

$$\{w_0 \mid w_0 \in V([w_0, x_1, y_0, z_1]_C), \{w, y\} \subset \mathbf{Z}_f, \{x, z\} \subset \mathbf{Z}_{v-f}\}$$

is even. This implies that the cardinality of the set

$$\{w_0 \mid w_0 \in V([w_0, x_1, y_1, z_1]_C), w \in \mathbf{Z}_f, \{x, y, z\} \subset \mathbf{Z}_{v-f}\}$$

is even. However as seen above, the cardinality of this set can be at most 1. Therefore, the cardinality of this set must be 0, and all arcs of the type (a_1, b_1) must be contained in blocks of the form $[w_1, x_1, y_1, z_1]_C$. However, the only such admissible blocks are $[0_1, 1_1, 2_1, 3_1]_C$ and $[3_1, 2_1, 1_1, 0_1]_C$, both of which are fixed under π and both of which contain 4 arcs of the form (a_1, b_1). Under our hypotheses, D_v contains 12 arcs of the form (a_1, b_1), therefore such a system cannot exist. ∎

Theorem 2.1 *An $f-$cyclic $4-$circuit decomposition of D_v exists if and only if $f \equiv 0$ or $1 \ (mod\ 4)$, $f \neq 4$, $v \equiv 0$ or $1 \ (mod\ 4)$, $v \neq 4$, and $v - f \geq 8$ in the case $f \equiv v \equiv 0 \ (mod\ 4)$.*

Proof. The fact that a $4-$circuit decomposition of D_v exists only if $v \equiv 0$ or 1 (mod 4), $v \neq 4$, along with Lemma 2.1, give the necessary congruence conditions on v and f. The necessity of $v \geq f + 8$ for $f \equiv v \equiv 0$ (mod 4) is given in Lemma 2.2. These conditions are shown to be sufficient in the following four cases.

Case 1. Suppose $f \equiv 0$ (mod 4), $f \neq 4$, $v \equiv 0$ (mod 4), $v \neq 4$, and $v \geq f + 8$. Say $v - f = 4t$. Consider the blocks:

$[0_1, i_1, (t + 2i - 1)_1, (t + i - 1)_1]_C$ for $i = 2, 3, \ldots, t - 1$,
$[(2i)_0, 0_1, (2i + 1)_0, 1_1]_C$ for $i = 1, 2, \ldots, f/2 - 1$.
$[0_1, t_1, (2t)_1, (3t)_1]_C, [0_1, 1_1, (2t)_1, (2t + 1)_1]_C$,
$[0_0, 1_1, (2t + 1)_1, (t + 1)_1]_C$, and $[1_0, 0_1, (2t + 1)_1, (2t)_1]_C$.

Case 2. Suppose $f \equiv 0$ (mod 4), $f \neq 4$, $v \equiv 1$ (mod 4) and $v \geq f + 8$. Say $v - f = 4t - 1$. Consider the blocks:

$[(2i)_0, 0_1, (2i + 1)_0, 1_1]_C$ for $i = 0, 1, \ldots, f/2 - 1$,
and the blocks for a cyclic $4-$circuit decomposition of D_{v-f} on the vertex set $\{0_1, 1_1, \ldots, (v - f - 1)_1\}$.

Case 3. Suppose $f \equiv 1 \pmod 4$ and $v \equiv 0 \pmod 4$, $v \neq 4$ and $v \geq f+8$. Say $v - f = 4t - 1$. Consider the blocks:

$[0_1, (1+2i)_1, (3+4i)_1, (2+2i)_1]_C$ for $i = 0, 1, \ldots, t-3$,

$[(3+2i)_0, 0_1, (4+2i)_0, 1_1]_C$ for $i = 0, 1, \ldots, (f-5)/2$,

$[0_0, 0_1, (2t-3)_1, (4t-3)_1]_C, [1_0, 0_1, (2t-2)_1, (4t-3)_1]_C$ and $[2_0, 0_1, (2t+1)_1, 4_1]_C$.

Case 4. Suppose $f \equiv 1 \pmod 4$ and $v \equiv 1 \pmod 4$ and $v \geq f+8$. Say $v - f = 4t$. Consider the blocks:

$[0_1, i_1, (t+2i)_1, (t+i)_1]_C$ for $i = 1, 2, \ldots, t-1$,

$[(2i-1)_0, 0_1, (2i)_0, 1_1]_C$ for $i = 1, 2, \ldots, (f-1)/2$,

$[0_1, t_1, (2t)_1, (3t)_1]_C$, and $[0_0, 0_1, (2t)_1, t_1]_C$.

In each case, these blocks, along with their images under the permutation $(0_0)(1_0)\cdots(f-1)_0(0_1, 1_1, \ldots, (v-f-1)_1)$ and the blocks for a 4–circuit decomposition of D_f on the vertex set $\{0_0, 1_0, \ldots, (f-1)_0\}$, form an f–cyclic 4–circuit decomposition of D_v. ∎

Lemma 2.3 *An f–cyclic X–decomposition of D_v satisfies the condition $v \geq 3f+1$.*

Proof. First, we observe that it is impossible for such a decomposition to contain a block of the form $[w_0, x_1, y_0, z_1]_X$. Applying π^{x-z} yields $[\pi^{x-z}(w_0), \pi^{x-z}(x_1), \pi^{x-z}(y_0), \pi^{x-z}(z_1)]_X = [w_0, \pi^{x-z}(x_1), y_0, x_1]_X$, a contradiction since these are distinct blocks which both contain the arc (w_0, x_1). Similarly, such a decomposition cannot contain blocks of the form $[w_1, x_0, y_1, z_0]_X$. Therefore by Lemma 2.1, for each fixed point w_0, we have $w_0 \in V(g_{w_0})$ for some g_{w_0} where $V(g_{w_0}) = \{w_0, x_1, y_1, z_1\}$. Let $S_{w_0} = \bigcup_{n \in Z} A(\pi^n(g_{w_0}))$ and

$$S = \bigcup_{\{w_0 | w_0 \in \{0_0, 1_0, \ldots, (f-1)_0\}\}} S_{w_0}.$$

Now, there are $(v-f)(v-f-1)$ arcs of the form (a_1, b_1) in $A(D_v)$ and there are $2f(v-f)$ arcs of this form in S. So it is necessary that $(v-f)(v-f-1) \geq 2f(v-f)$, or that $v \geq 3f+1$. ∎

Theorem 2.2 *An f–cyclic X–decomposition of D_v exists if and only if $v \geq 3f+1$ and <u>either</u> $f \equiv 0$ (mod 4) and $v \equiv 1$ (mod 4), $v \neq 5$, <u>or</u> $f \equiv 1$ (mod 4), $f \neq 5$, and $v \equiv 0$ (mod 4).*

Proof. As seen in the proof of Lemma 2.3, each block of such a decomposition must be of one of the following forms: $[w_0, x_0, y_0, z_0]_X$, $[w_1, x_0, y_1, z_1]_X$, $[w_1, x_1, y_0, z_1]_X$ or $[w_1, x_1, y_1, z_1]_X$. Now, the cardinality of the sets $\{\pi^n([w_1, x_0, y_1, z_1]_X) \mid n \in \mathbf{Z}\}, \{\pi^n([w_1, x_1, y_0, z_1]_X) \mid n \in \mathbf{Z}\}$ and $\{\pi^n([w_1, x_1, y_1, z_1]_X) \mid n \in \mathbf{Z}\}$ are each $(v-f)$. Since each of these blocks contains an even number of arcs of the type (a_1, b_1), it must be that the total number of such arcs is an even multiple of $(v-f)$. However, there are $(v-f)(v-f-1)$ arcs of this type in $A(D_v)$, and so it is not possible that $f \equiv v \pmod 4$. This condition, along with Lemmas 2.1 and 2.3 and the conditions for the existence of a X–decomposition of D_v gives the necessary conditions for the existence of a f–cyclic X–decomposition of D_v. We now establish sufficiency in the following four cases:

Case 1. Suppose $f \equiv 1 \pmod 4$, $f \neq 5$, $v \equiv 0 \pmod 4$, $v - f \equiv 7 \pmod 8$, and

$v \geq 3f+1$. Say $v-f=8t-1$. Consider the blocks:

$[0_1,(1+i)_1,(6t+2i)_1,(2t+i)_1]_X$ for $i=(f-1)/2,(f-1)/2+1,\ldots,t-1$ (omit if $t<(f+1)/2$,

$[0_1,(t+1+i)_1,(8t+2i)_1,(5t-1+i)_1]_X$ for $i=\max\{0,(f-1)/2-t\},\max\{0,(f-1)/2-t\}+1,\ldots,t-2$ (omit if $2t<(f+3)/2$),

$[0_1,(1+i)_0,(6t+2i)_1,(2t+i)_1]_X$ for $i=0,1,\ldots,\min\{t-1,(f-1)/2-1\}$,

$[0_1,(\min\{t+1,(f-1)/2+1\}+i)_0,(2t+1)_1,(1+i)_1]_X$ for $i=0,1,\ldots,\min\{t-1,(f-1)/2-1\}$,

$[0_1,(2t+1+i)_0,(8t+2i)_1,(5t-1+i)_1]_X$ for $i=0,1,\ldots,(f-1)/2-t-1$ (omit if $(f-1)/2-t<1$),

$[0_1,((f-1)/2+t+1+i)_0,(2t+1)_1,(t+1+i)_1]_X$ for $i=0,1,\ldots,(f-1)/2-t-1$ (omit if $(f-1)/2-t<1$), and

$[(6t-1)_1,0_0,0_1,(6t-2)_1]_X$.

Case 2. Suppose $f\equiv 1 \pmod 4$, $f\neq 5$, $v\equiv 0 \pmod 4$, $v-f\equiv 3 \pmod 8$, and $v\geq 3f+1$. Say $v-f=8t+3$. Consider the blocks:

$[0_1,(1+i)_1,(6t+4+2i)_1,(2t+1+i)_1]_X$ for $i=(f-1)/4,(f-1)/4+1,\ldots,t-1$,

$[0_1,(t+1+i)_1,(8t+4+2i)_1,(5t+1+i)_1]_X$ for $i=(f-1)/4,(f-1)/4+1,\ldots,t-1$,

$[0_1,(1+i)_0,(6t+4+2i)_1,(2t+1+i)_1]_X$ for $i=0,1,\ldots,(f-1)/4-1$,

$[0_1,((f-1)/4+1+i)_0,(1+2i)_1,(5t+1+i)_1]_X$ for $i=0,1,\ldots,(f-1)/4-1$,

$[0_1,((f-1)/2+1+i)_0,(2t+1)_1,(1+i)_1]_X$ for $i=0,1,\ldots,(f-1)/4-1$,

$[0_1,(3(f-1)/4+1+i)_0,(2t+1)_1,(t+1+i)_1]_X$ for $i=0,1,\ldots,(f-1)/4-1$, and

$[1_1,0_0,0_1,(6t+2)_1]_X$.

Case 3. Suppose $f\equiv 0 \pmod 4$, $v\equiv 1 \pmod 4$, $v\neq 5$, $v-f\equiv 1 \pmod 8$, and $v\geq 3f+1$. Say $v-f=8t+1$. Consider the blocks:

$[0_1,(1+i)_1,(6t+2+2i)_1,(2t+1+i)_1]_X$ for $i=f/4,f/4+1,\ldots,t-1$,

$[0_1,(t+1+i)_1,(1+2i)_1,(5t+1+i)_1]_X$ for $i=f/4,f/4+1,\ldots,t-1$,

$[0_1,i_0,(6t+2+2i)_1,(2t+1+i)_1]_X$ for $i=0,1,\ldots,f/4-1$,

$[0_1,(f/4+i)_0,(1+2i)_1,(5t+1+i)_1]_X$ for $i=0,1,\ldots,f/4-1$,

$[0_1,(f/2+i)_0,(2t+1)_1,(1+i)_1]_X$ for $i=0,1,\ldots,f/4-1$, and

$[0_1,(3f/4+i)_0,(2t+1)_1,(t+1+i)_1]_X$ for $i=0,1,\ldots,f/4-1$.

Case 4. Suppose $f\equiv 0 \pmod 4$, $v\equiv 1 \pmod 4$, $v\neq 5$, $v-f\equiv 5 \pmod 8$, and $v\geq 3f+1$. Say $v-f=8t+5$. Consider the blocks:

$[0_1,(2+i)_1,1_1,(2t+5+2i)_1]_X$ for $i=f/2,f/4+1,\ldots,t$ (omit if $t<f/2$),

$[0_1,(t+3+i)_1,1_1,(4t+8+2i)_1]_X$ for $i=\max\{0,f/2-1-t\},\max\{0,f/2-1-t\}+1,\ldots,t-2$ (omit if $2t<f/2+1$),

$[0_1,i_0,(3+2i)_1,(2+i)_1]_X$ for $i=0,1,\ldots,\min\{t,f/2-1\}$,

$[0_1,(\min\{t+1,f/2\}+i)_0,1_1,(2t+5+2i)_1]_X$ for $i=0,1,\ldots,\min\{t,f/2-1\}$,

$[0_1,(2t+2+i)_0,(2t+5+2i)_1,(t+3+i)_1]_X$ for $i=0,1,\ldots,f/2-t-2$ (omit if $f/2-t<2$),

$[0_1,(f/2+t+1+i)_0,1_1,(4t+8+2i)_1]_X$ for $i=0,1,\ldots,f/2-t-2$ (omit if $f/2-t<2$), and

$[0_1,1_1,(2t+3)_1,(4t+6)_1]_X$.

In each case, these blocks, along with their images under the permutation $(0_0)(1_0)\cdots(f-1)_0(0_1,1_1,\ldots,(v-f-1)_1)$ and the blocks for a X−decomposition of D_f on the vertex set $\{0_0,1_0,\ldots,(f-1)_0\}$, form an f−cyclic X−decomposition of D_v. ∎

Theorem 2.3 *An f−cyclic Y−decomposition of D_v exists if and only if* <u>*either*</u> $f\equiv 0$

(mod 4), $f \neq 4$, and $v \equiv 1$ (mod 4), $v \neq 5$ or $f \equiv 1$ (mod 4), $f \neq 5$, and $v \equiv 0$ (mod 4), $v \neq 4$.

Proof. By Lemma 2.1, each arc of the form (a_1, b_1) must be contained in a block of one of the following forms: $[w_1, x_0, y_1, z_1]_Y$ or $[w_1, x_1, y_1, z_1]_Y$. Now, the cardinality of the sets $\{\pi^n([w_1, x_0, y_1, z_1]_Y) \mid n \in \mathbf{Z}\}$ and $\{\pi^n([w_1, x_1, y_1, z_1]_Y) \mid n \in \mathbf{Z}\}$ are both $(v-f)$. Since each of these blocks contains an even number of arcs of the form (a_1, b_1), it must be that the total number of such arcs is an even multiple of $(v-f)$. However, there are $(v-f)(v-f-1)$ arcs of this form in $A(D_v)$, and so it is not possible that $f \equiv v$ (mod 4). This condition, along with Lemma 2.1 and the conditions for the existence of a $Y-$decomposition of D_v gives the necessary conditions for the existence of a $f-$cyclic $Y-$decomposition of D_v. We now establish sufficiency in the following two cases:
Case 1. Suppose $f \equiv 1$ (mod 4), $f \neq 5$, and $v \equiv 0$ (mod 4), $v \neq 4$. Then $v - f \equiv 3$ (mod 4), say $v - f = 4t - 1$. Consider the blocks:
$[0_1, (1+i)_1, (4t-3)_1, (2t-1+i)_1]_Y$ for $i = (f-1)/2, (f-1)/2+1, \ldots, t-2$,
$[0_1, (1+i)_0, (4t-3)_1, (2t-1+i)_1]_Y$ for $i = 0, 1, \ldots, (f-1)/2-1$,
$[0_1, ((f-1)/2+1+i)_0, (4t-3)_1, (1+i)_1]_Y$ for $i = 0, 1, \ldots, (f-1)/2-1$, and
$[1_1, 0_0, (4t-3)_1, 0_1]_Y$.
Case 2. Suppose $f \equiv 0$ (mod 4), $f \neq 4$, and $v \equiv 1$ (mod 4), $v \neq 5$. Then $v - f \equiv 1$ (mod 4), say $v - f = 4t + 1$. Consider the blocks:
$[0_1, (1+i)_1, (4t-1)_1, (2t+1+i)_1]_Y$ for $i = f/2-1, f/2, \ldots, t-2$,
$[0_1, i_0, (4t-1)_1, (2t+1+i)_1]_Y$ for $i = 0, 1, \ldots, f/2-2$,
$[0_1, (f/2-1+i)_0, (4t-1)_1, (1+i)_1]_Y$ for $i = 0, 1, \ldots, f/2-2$, and
$[0_1, (2t-1)_1, (2t-2)_1, (4t-1)_1]_Y$ and $[0_1, (f-2)_0, 1_1, (f-1)_0]_Y$.
In either case, these blocks, along with their images under the permutation $(0_0)(1_0)\cdots(f-1)_0(0_1, 1_1, \ldots, (v-f-1)_1)$ and the blocks for a $Y-$decomposition of D_f on the vertex set $\{0_0, 1_0, \ldots, (f-1)_0\}$, form an $f-$cyclic $Y-$decomposition of D_v. ∎

Theorem 2.4 *An $f-$cyclic $Z-$decomposition of D_v does not exist.*

Proof. Suppose that such a system exists. We observe that the system can contain no blocks of the form $[w_0, x_1, y_1, z_1]_Z$ or $[x_1, w_0, y_1, z_1]_Z$, for applying π^{x-z} to such blocks leads to a contradiction, as in the proof of Lemma 2.3. So all arcs of the form (a_1, b_1) must be contained in blocks of the form $[w_1, x_1, y_1, z_1]_Z$. Therefore, there is a cyclic subsystem of the given system of order $(v-f)$. So $v - f \equiv 1$ (mod 4). But by Lemma 2.1, $f \equiv 1$ (mod 4) and so $v \equiv 2$ (mod 4), a contradiction. ∎

References

[1] C. J. Cho, Rotational Mendelsohn triple systems, *Kyungpook Math. J.* **26**(1) (1986), 5-9.

[2] C. J. Cho, Y. C. Chae and S. G. Hwang, Rotational directed triple systems, *J. Korean Math. Soc.* **24**(2) (1987), 133-142.

[3] G. D. Coker, R. Gardner and R. Johnson, Decompositions of the complete digraph and the complete graph which admit certain automorphisms, *Congressus Numerantium* **108** (1995), 64-84.

[4] C. Colbourn and M. Colbourn, Disjoint cyclic Mendelsohn triple systems, *Ars Combinatoria* **11** (1981), 3-8.

[5] M. Colbourn and C. Colbourn, The analysis of directed triple systems by refinement, *Annals of Discrete Math.* **15** (1982), 97-103.

[6] F. Harary, W. Wallis, and K. Heinrich, Decompositions of complete symmetric digraphs into the four oriented quadrilaterals, *Combinatorial Mathematics* in Lecture Notes in Mathematics #686, Springer-Verlag, 1978.

[7] D. G. Hoffman, Cyclic embeddings of semi-symmetric quasigroups and Mendelsohn triple systems, *Ars Combinatoria*, to appear.

[8] S. Hung and N. Mendelsohn, Directed triple systems, *J. Combin. Theory Series A* **14** (1973), 310-318.

[9] N. Mendelsohn, A natural generalization of Steiner triple systems, *Computers in Number Theory*, eds. A. O. Atkin and B. Birch, Academic Press, London, 1971.

[10] B. Micale and M. Pennisi, Cyclic decompositions of complete symmetric digraphs into the four oriented quadrilaterals, *Utilitas Mathematica*, to appear.

[11] B. Micale and M. Pennisi, Cyclic Mendelsohn quadruple systems, *Ars Combinatoria* **35** (1993), 225-236.

[12] B. Micale and M. Pennisi, The spectrum of $d-$cyclic oriented triple systems, Ars Combinatoria, to appear.

[13] M. Pennisi, On the rotational Mendelsohn designs, *J. Combinatorics, Information and System Sciences*, to appear.

[14] J. Schönheim, Partition of the edges of the directed complete graph into 4-cycles, *Discrete Math.* **11** (1975), 67-70.