Reverse Directed Triple Systems

Rebecca Calahan-Zijlstra
Department of Mathematics and Statistics
Middle Tennessee State University
Murfreesboro, Tennessee 37312

Robert B. Gardner*
Department of Mathematics
East Tennessee State University
Johnson City, Tennessee 37614

ABSTRACT. A directed triple system of order v and index λ, denoted $DTS_\lambda(v)$, is said to be *reverse* if it admits an automorphism consisting of $v/2$ transpositions when v is even, or a fixed point and $(v-1)/2$ transpositions when v is odd. We give necessary and sufficient conditions for the existence of a reverse $DTS_\lambda(v)$ for all $\lambda \geq 1$.

1 Introduction

A *directed triple system* of order v and index λ, denoted $DTS_\lambda(v)$, is a v-element set X, of points, together with a set B, of ordered triples of elements of X, called *blocks*, such that any ordered pair of points of X occurs in exactly λ blocks of B. The notation $[x, y, z]$ will be used for the block containing the ordered pairs (x, y), (x, z), and (y, z). Hung and Mendelsohn [6] introduced directed triple systems as a generalization of Steiner triple systems and showed that a $DTS_1(v)$ exists if and only if $v \equiv 0$ or $1 \pmod 3$. Seberry and Skillicorn [8] proved that a $DTS_\lambda(v)$ exists if and only if $\lambda v(v-1) \equiv 0 \pmod 3$, $v \neq 2$.

An *automorphism* of a $DTS_\lambda(v)$ is a permutation of X which fixes B. The *orbit* of a block under an automorphism π is the image of the block under the powers of π. A collection of blocks β is said to be a *collection*

*Research supported by ETSU under RDC grant #94-105/M.

of base blocks for a $DTS_\lambda(v)$ under the permutation π if the orbits of the blocks of β produce the $DTS_\lambda(v)$.

Several types of automorphisms have been explored in connection with the problem of determining the values v for which there are certain types of block designs of order v admitting the automorphism. In particular, a *cyclic* $DTS_\lambda(v)$ admits an automorphism consisting of a single cycle of length v and exists if and only if [2, 4]:

1. $\lambda \equiv 0 \pmod 6$ and $v \neq 2$, or

2. $\lambda \equiv 1$ or $5 \pmod 6$ and $v \equiv 1$, 4 or $7 \pmod{12}$, or

3. $\lambda \equiv 2$ or $4 \pmod 6$ and $v \equiv 1 \pmod 3$, or

4. $\lambda \equiv 3 \pmod 6$ and $v \equiv 0$, 1 or $3 \pmod 4$.

A $DTS_\lambda(v)$ which admits an automorphism consisting of a fixed point and k cycles of length $(v-1)/k$ is said to be *k-rotational*. A k-rotational $DTS_1(v)$ exists if and only if $kv \equiv 0 \pmod 3$ and $v \equiv 1 \pmod k$ [1]. A 1-rotational $DTS_\lambda(v)$ exists if and only if $\lambda v \equiv 0 \pmod 3$ and $v \geq 3$ [3]. These two results, along with the observation that $\lambda k v \equiv 0 \pmod 3$ is a necessary condition for the existence of a k-rotational $DTS_\lambda(v)$, yield:

Corollary 1.1. *A k-rotational $DTS_\lambda(v)$ exists if and only if $\lambda kv \equiv 0$ (mod 3), $v \equiv 1$ (mod k) and $v \geq 3$.*

Steiner triple systems, denoted STS, have been extensively explored in connection with these types of questions. In particular, a *reverse* $STS(v)$ admits an automorphism consisting of a fixed point and $(v-1)/2$ transpositions. A reverse $STS(v)$ exists if and only if $v \equiv 1$, 3, 9 or 19 (mod 24) [5, 7, 9, 10]. With this result as motivation, we define a *reverse* $DTS_\lambda(v)$ to be one admitting an automorphism consisting of a fixed point and $(v-1)/2$ transpositions if v is odd, or $v/2$ transpositions if v is even. The purpose of this paper is to use the above mentioned results along with some new constructions to give necessary and sufficient conditions for the existence of a reverse $DTS_\lambda(v)$ for all $\lambda \geq 1$. We will take advantage of the fact that if there exists a $DTS_{\lambda_1}(v)$ and a $DTS_{\lambda_2}(v)$ both of which admit π as an automorphism, then there exists a $DTS_{\lambda_1+\lambda_2}(v)$ admitting π as an automorphism.

2 Reverse Directed Triple Systems With $\lambda = 1$

In this section and the next section we will deal with reverse $DTS_\lambda(v)$ on the set $X = \{a, b\} \times \mathbf{Z}_{v/2}$ admitting the automorphism $\pi = (a_0, b_0)(a_1, b_1) \cdots (a_{v/2-1}, b_{v/2-1})$. We represent the ordered pair (x, y) as x_y.

Lemma 2.1. *If a reverse $DTS_\lambda(v)$ exists where v is even, then $\lambda v(v-4) \equiv 0$ (mod 24).*

Proof: Each block of such a $DTS_\lambda(v)$ must be of one of the following forms:

1. $[a_i, a_j, a_k]$ or $[b_i, b_j, b_k]$ where i, j, k are distinct,
2. $[a_i, b_j, b_k]$ or $[b_i, a_j, a_k]$ where $j \neq k$,
3. $[a_i, b_j, a_k]$ or $[b_i, a_j, b_k]$ where $i \neq k$, or
4. $[a_i, a_j, b_k]$ or $[b_i, b_j, a_k]$ where $i \neq j$.

Let r be the number of blocks of type 1, s the number of type 2, t the number of type 3, and u the number of type 4. Notice that r, s, t and u are all even. The number of blocks in a $DTS_\lambda(v)$ is $\lambda v(v-1)/3$ so $r+s+t+u = \lambda v(v-1)/3$. In this $DTS_\lambda(v)$ there is a total of $\lambda v(v-2)/2$ pairs of the form (α_i, α_j) where $\alpha \in \{a, b\}$, $i \neq j$. Blocks of the first type each contain 3 such pairs, blocks of the second, third and fourth types each contain 1 such pair. So $3r + s + t + u = \lambda v(v-2)/2$. So $r = \lambda v(v-4)/12$ where r is even. □

The conditions for the existence of a $DTS_1(v)$ along with Lemma 2.1 imply that the necessary conditions for the existence of a reverse $DTS_1(v)$ are $v \equiv 0, 1, 3, 4, 7$, or 9 (mod 12). We now show that these necessary conditions are sufficient.

Theorem 2.1. *A reverse $DTS_1(v)$ exists if and only if $v \equiv 0, 1, 3, 4, 7$, or 9 (mod 12).*

Proof: For sufficiency, we present five cases.

Case 1. Suppose that $v \equiv 1$ or 3 (mod 6). Then there exists a $(v-1)/2$–rotational $DTS_1(v)$ by Corollary 1.1. This $DTS_1(v)$ is clearly also reverse.

Case 2. Suppose that $v \equiv 4$ (mod 12). Then there exists a cyclic $DTS_1(v)$ admitting an automorphism α which consists of a single cycle of length v. The automorphism $\alpha^{v/2}$ consists of $v/2$ transpositions and therefore this $DTS_1(v)$ is also reverse.

Case 3a. Suppose that $v = 24$. Let α be the permutation $(a_0, a_1, \cdots, a_9, b_0, b_1, \cdots, b_9)\,(a_{10}, a_{11}, b_{10}, b_{11})$. Consider the blocks:

$[\alpha^j(a_{10}), \alpha^j(a_{11}), \alpha^j(b_{11})]$ for $j = 0, 1, 2, 3$, and

$[\alpha^j(a_{10}), \alpha^j(a_1), \alpha^j(a_0)]$, $[\alpha^j(a_2), \alpha^j(a_0), \alpha^j(a_{10})]$, $[\alpha^j(a_1), \alpha^j(a_{10}), \alpha^j(b_8)]$, $[\alpha^j(a_3), \alpha^j(a_{10}), \alpha^j(b_9)]$, $[\alpha^j(a_0), \alpha^j(a_1), \alpha^j(a_8)]$, $[\alpha^j(a_0), \alpha^j(a_2), \alpha^j(b_5)]$, $[\alpha^j(a_0), \alpha^j(a_3), \alpha^j(b_2)]$, $[\alpha^j(a_0), \alpha^j(a_4), \alpha^j(b_4)]$, $[\alpha^j(a_0), \alpha^j(a_5), \alpha^j(b_1)]$ for $j = 0, 1, \ldots, 19$.

These blocks form a collection of base blocks for a reverse $DTS_1(24)$ under π.

Case 3b. Suppose that $v \equiv 0 \pmod{24}$, $v \neq 24$. Let $v = 24t$, $t \geq 2$, and let α be the permutation $(a_0, a_1, \cdots, a_{12t-3}, b_0, b_1, \cdots, b_{12t-3})(a_{12t-2}, a_{12t-1}, b_{12t-2}, b_{12t-1})$. Consider the blocks:

$[\alpha^j(a_{12t-2}), \alpha^j(a_{12t-1}), \alpha^j(b_{12t-1})]$ for $j = 0, 1, 2, 3$,

$[\alpha^j(a_{12t-2}), \alpha^j(a_1), \alpha^j(a_0)]$ and $[\alpha^j(a_2), \alpha^j(a_0), \alpha^j(a_{12t-2})]$ for $j = 0, 1, \ldots, 24t-5$,

$[\alpha^j(a_1), \alpha^j(a_{12t-2}), \alpha^j(b_{12t-4})]$ for $j = 0, 1, \ldots, 24t-5$,

$[\alpha^j(a_3), \alpha^j(a_{12t-2}), \alpha^j(b_{12t-3})]$ for $j = 0, 1, \ldots, 24t-5$,

$[\alpha^j(a_0), \alpha^j(a_1), \alpha^j(a_{10t-2})]$ and $[\alpha^j(a_0), \alpha^j(a_{8t-3}), \alpha^j(b_{8t-5})]$ for $j = 0, 1, \ldots, 24t-5$,

$[\alpha^j(a_0), \alpha^j(a_{4t-3}), \alpha^j(b_{4t-4})]$ for $j = 0, 1, \ldots, 24t-5$,

$[\alpha^j(a_0), \alpha^j(a_{8t-4-2i}), \alpha^j(b_{12t-7-i})]$ for $i = 0, 1, \ldots, 4t-3$ and $j = 0, 1, \ldots, 24t-5$,

$[\alpha^j(a_0), \alpha^j(a_{8t-5-2i}), \alpha^j(b_{4t-5-i})]$ for $i = 0, 1, \ldots, 2t-2$ and $j = 0, 1, \ldots, 24t-5$,

$[\alpha^j(a_0), \alpha^j(a_{4t-5-2i}), \alpha^j(b_{2t-4-i})]$ for $i = 0, 1, \ldots, 2t-4$ and $j = 0, 1, \ldots, 24t-5$.

These blocks form a collection of base blocks for a reverse $DTS_1(v)$ under π.

Case 4. Suppose that $v \equiv 12 \pmod{48}$. Let $v = 48t + 12$. Consider the blocks:

$[a_i, a_{8t+2+i}, a_{16t+4+i}]$ and $[a_{16t+4+i}, a_{8t+2+i}, a_i]$ for $i = 0, 1, \ldots, 8t+1$,

$[a_i, a_{10t+2+i}, a_{14t+2+i}]$ for $i = 0, 1, \ldots, 24t+5$ (omit if $t = 0$),

$[a_i, a_{6t-2j+i}, a_{6t+2+2j+i}]$ for $i = 0, 1, \ldots, 24t+5$ and $j = 0, 1, \ldots, t-1$ (omit if $t = 0$),

$[a_i, a_{10t-2j+i}, a_{10t+4+2j+i}]$ for $i = 0, 1, \ldots, 24t+5$ and $j = 0, 1, \ldots, t-2$ (omit if $t = 0$),

$[a_i, a_{14t+4+2j+i}, a_{14t-2j+i}]$ for $i = 0, 1, \ldots, 24t+5$ and $j = 0, 1, \ldots, t-1$ (omit if $t = 0$),

$[a_i, a_{18t+6+2j+i}, a_{18t+4-2j+i}]$ for $i = 0, 1, \ldots, 24t+5$ and $j = 0, 1, \ldots, t-1$ (omit if $t = 0$),

$[a_i, b_{6t+1-j+i}, b_{6t+2+j+i}]$ for $i = 0, 1, \ldots, 24t+5$ and $j = 0, 1, \ldots, 6t+1$,

$[a_i, b_{18t+5+j+i}, b_{18t+4-j+i}]$ for $i = 0, 1, \ldots, 24t+5$ and $j = 0, 1, \ldots, 6t$.

These blocks form a collection of base blocks for a reverse $DTS_1(v)$ under π.

Case 5. Suppose that $v \equiv 36 \pmod{48}$. Let $v = 48t + 36$. Consider the blocks:

$[a_i, a_{8t+6+i}, a_{16t+12+i}]$ and $[a_{16t+12+i}, a_{8t+6+i}, a_i]$ for $i = 0, 1, \ldots, 8t+5$,

$[a_i, a_{6t+5+i}, a_{10t+8+i}]$ for $i = 0, 1, \ldots, 24t+17$,

$[a_i, a_{6t+3-j+i}, a_{6t+6+j+i}]$ for $i = 0, 1, \ldots, 24t+17$ and $j = 0, 1, \ldots, 2t-1$ (omit if $t = 0$),

$[a_i, a_{10t+7-j+i}, a_{10t+9+j+i}]$ for $i = 0, 1, \ldots, 24t+17$ and $j = 0, 1, \ldots, 2t$,

$[a_i, b_{12t+8+i}, b_{18t+12+i}]$ for $i = 0, 1, \ldots, 24t+17$,

$[a_i, b_{22t+15+i}, b_{22t+16+i}]$ for $i = 0, 1, \ldots, 24t+17$,

$[a_i, b_{6t+4+j+i}, b_{6t+3-j+i}]$ for $i = 0, 1, \ldots, 24t+17$ and $j = 0, 1, \ldots, 6t+3$,

$[a_i, b_{18t+13+j+i}, b_{18t+11-j+i}]$ for $i = 0, 1, \ldots, 24t+17$ and $j = 0, 1, \ldots, 4t+1$,

$[a_i, b_{22t+17+j+i}, b_{14t+9-j+i}]$ for $i = 0, 1, \ldots, 24t+17$ and $j = 0, 1, \ldots, 2t$.

These blocks form a collection of base blocks for a reverse $DTS_1(v)$ under π. □

3 Reverse Directed Triple Systems With $\lambda > 1$

Finally, we give necessary and sufficient conditions for the existence of a reverse $DTS_\lambda(v)$ where $\lambda > 1$.

Theorem 3.1. *A reverse $DTS_\lambda(v)$, where v is odd, exists if and only if $\lambda v(v-1) \equiv 0 \pmod 3$. A reverse $DTS_\lambda(v)$, where v is even, exists if and only if $\lambda v(v-1) \equiv 0 \pmod 3$ and $\lambda v(v-4) \equiv 0 \pmod{24}$, $v \neq 2$.*

Proof: The necessary conditions follow from the conditions for the existence of a $DTS_\lambda(v)$ along with Lemma 2.1. We show sufficiency in the following cases.

Case 1. Suppose that $v \equiv 0, 1, 3, 4, 7$, or $9 \pmod{12}$. Then there exists a reverse $DTS_1(v)$ by Theorem 2.1. Therefore there exists a reverse $DTS_\lambda(v)$ for all $\lambda \geq 1$.

Case 2. Suppose that $v \equiv 2 \pmod{12}$. Then it is necessary that $\lambda \equiv 0 \pmod{6}$. In this case, there is a $DTS_\lambda(v)$ admitting a cyclic automorphism α. The automorphism $\alpha^{v/2}$ consists of $v/2$ transpositions and therefore this $DTS_\lambda(v)$ is also reverse.
Case 3. Suppose that $v \equiv 5 \pmod{6}$. Then there exists a $(v-1)/2$-rotational $DTS_\lambda(v)$ by Corollary 1.1. This $DTS_\lambda(v)$ is clearly reverse.
Case 4a. Suppose that $v = 6$. Consider the blocks:

$$[a_0, b_1, a_2],\ [a_1, b_2, a_0],\ [a_2, b_0, a_1],\ [a_1, a_0, b_0],\ [a_2, a_1, b_1],$$

$$[a_2, a_0, b_2],\ [b_1, a_0, a_1],\ [b_2, a_1, a_2],\ [b_0, a_2, a_0], \text{ and } [a_0, a_1, a_2].$$

These blocks form a collection of base blocks for a reverse $DTS_2(6)$. Therefore there exists a reverse $DTS_\lambda(6)$ for all $\lambda \equiv 0 \pmod{2}$.
Case 4b. Suppose that $v \equiv 6 \pmod{24}$, $v \neq 6$, say $v = 24t + 6$, $t \geq 1$. Consider the blocks:

$[a_i, a_{6t-j+i}, a_{6t+1+j+i}]$ for $i = 0, 1, \ldots, 12t + 2$ and $j = 0, 1, \ldots, t-1$,

$[a_i, a_{5t-j+i}, a_{7t+3+j+i}]$ for $i = 0, 1, \ldots, 12t + 2$ and $j = 0, 1, \ldots, t-2$ (omit if $t = 1$),

$[a_i, a_{2+2j+i}, a_{10t+3+j+i}]$ for $i = 0, 1, \ldots, 12t + 2$ and $j = 0, 1, \ldots, 2t-2$,

$[a_i, a_{7t+2+i}, a_{7t+1+i}]$ for $i = 0, 1, \ldots, 12t + 2$,

$[a_i, a_{4t+1+i}, a_{8t+2+i}]$ and $[a_i, a_{8t+2+i}, a_{4t+1+i}]$ for $i = 0, 1, \ldots, 8t + 1$,

$[a_i, b_{10t+3+j+i}, a_{8t+4+2j+i}]$ for $i = 0, 1, \ldots, 12t+2$ and $j = 0, 1, \ldots, 4t-1$,

$[a_i, b_{2t+1+j+i}, a_{4t+2+2j+i}]$ for $i = 0, 1, \ldots, 12t+2$ and $j = 0, 1, \ldots, 2t-1$,

$[a_i, b_{4t+1+j+i}, a_{8t+3+2j+i}]$ for $i = 0, 1, \ldots, 12t+2$ and $j = 0, 1, \ldots, 2t-1$,

$[a_i, b_{6t+2+j+i}, a_{2+2j+i}]$ for $i = 0, 1, \ldots, 12t + 2$ and $j = 0, 1, \ldots, 2t-1$,

$[a_i, b_{8t+3+j+i}, a_{4t+3+2j+i}]$ for $i = 0, 1, \ldots, 12t+2$ and $j = 0, 1, \ldots, 2t-1$,

$[a_i, b_{10t+3+i}, b_{2t+i}]$ for $i = 0, 1, \ldots, 12t + 2$,

$[a_i, b_{4t+1+i}, b_{6t+2+i}]$ for $i = 0, 1, \ldots, 12t + 2$, and

$[a_i, b_{8t+2+i}, b_{6t+1+i}]$ for $i = 0, 1, \ldots, 12t + 2$.

These blocks form a collection of base blocks for a reverse $DTS_2(v)$. Therefore there exists a reverse $DTS_\lambda(v)$ for all $\lambda \equiv 0 \pmod{2}$.
Case 5. Suppose that $v \equiv 8 \pmod{12}$. Then it is necessary that $\lambda \equiv 0 \pmod{3}$. Under these conditions, there is a cyclic $DTS_\lambda(v)$ and this $DTS_\lambda(v)$ is also reverse by the argument of Case 2.

Case 6. Suppose that $v \equiv 10 \pmod{12}$. Then it is necessary that $\lambda \equiv 0 \pmod 2$. Under these conditions, there is a cyclic $DTS_\lambda(v)$ and this $DTS_\lambda(v)$ is also reverse by the argument of Case 2.
Case 7a. Suppose that $v = 18$. Consider the blocks:

$[a_i, a_{3+i}, a_{6+i}]$ and $[a_i, a_{6+i}, a_{3+i}]$ for $i = 0, 1, 2, 3, 4, 5$, along with

$[a_i, a_{7+i}, a_{8+i}]$, $[a_i, b_i, b_{1+i}]$, $[a_i, b_{1+i}, b_{3+i}]$, $[a_i, b_{2+i}, b_{6+i}]$, $[a_i, b_{3+i}, b_{8+i}]$, $[a_i, b_{5+i}, b_{4+i}]$, $[a_i, b_{8+i}, b_{6+i}]$, $[a_i, b_i, b_{4+i}]$, $[a_i, b_{5+i}, b_{7+i}]$, and $[a_i, b_{2+i}, b_{7+i}]$ for $i = 0, 1, \ldots, 8$.

These blocks form a collection of base blocks for a reverse $DTS_2(18)$. Therefore there exists a reverse $DTS_\lambda(18)$ for all $\lambda \equiv 0 \pmod 2$.
Case 7b. Suppose that $v \equiv 18 \pmod{24}$, $v \neq 18$, say $v = 24t + 18$, $t \geq 1$. Consider the blocks:

$[a_i, a_{6t+3-j+i}, a_{6t+5+j+i}]$ for $i = 0, 1, \ldots, 12t + 8$ and $j = 0, 1, \ldots, t - 1$,

$[a_i, a_{5t+3-j+i}, a_{7t+7+j+i}]$ for $i = 0, 1, \ldots, 12t + 8$ and $j = 0, 1, \ldots, t - 2$ (omit if $t = 1$),

$[a_i, a_{10t+6-j+i}, a_{10t+9+j+i}]$ for $i = 0, 1, \ldots, 12t+8$ and $j = 0, 1, \ldots, 2t-1$,

$[a_i, a_{7t+5+i}, a_{7t+6+i}]$ for $i = 0, 1, \ldots, 12t + 8$,

$[a_i, a_{6t+4+i}, a_{10t+8+i}]$ for $i = 0, 1, \ldots, 12t + 8$,

$[a_i, a_{4t+3+i}, a_{8t+6+i}]$ and $[a_i, a_{8t+6+i}, a_{4t+3+i}]$ for $i = 0, 1, \ldots, 8t + 5$,

$[a_i, b_{10t+8+j+i}, a_{8t+8+2j+i}]$ for $i = 0, 1, \ldots, 12t+8$ and $j = 0, 1, \ldots, 4t+1$,

$[a_i, b_{2t+2+j+i}, a_{4t+4+2j+i}]$ for $i = 0, 1, \ldots, 12t + 8$ and $j = 0, 1, \ldots, 2t$,

$[a_i, b_{4t+3+j+i}, a_{8t+7+2j+i}]$ for $i = 0, 1, \ldots, 12t + 8$ and $j = 0, 1, \ldots, 2t$,

$[a_i, b_{6t+5+j+i}, a_{2+2j+i}]$ for $i = 0, 1, \ldots, 12t + 8$ and $j = 0, 1, \ldots, 2t$,

$[a_i, b_{8t+7+j+i}, a_{4t+5+2j+i}]$ for $i = 0, 1, \ldots, 12t + 8$ and $j = 0, 1, \ldots, 2t$,

$[a_i, b_{10t+8+i}, b_{2t+1+i}]$ for $i = 0, 1, \ldots, 12t + 8$,

$[a_i, b_{8t+6+i}, b_{6t+4+i}]$ for $i = 0, 1, \ldots, 12t + 8$, and

$[a_i, b_{4t+3+i}, b_{6t+5+i}]$ for $i = 0, 1, \ldots, 12t + 8$.

These blocks form a collection of base blocks for a reverse $DTS_2(v)$. Therefore there exists a reverse $DTS_\lambda(v)$ for all $\lambda \equiv 0 \pmod 2$. □

Theorem 3.1 gives a complete classification of reverse directed triple systems.

References

[1] C. J. Cho, Y. Chae and S. G. Hwang, Rotational directed triple systems, *J. Korean Math. Soc.* **24** (1987), 133–142.

[2] C. J. Cho, Y. Han and S. Kang, Cyclic directed triple systems, *J. Korean Math. Soc.* **23** (1986), 117–125.

[3] C. J. Colbourn, Automorphisms of directed triple systems, *Bull. Austral. Math. Soc.* **43** (1991), 257–264.

[4] M. J. Colbourn and C. J. Colbourn, The analysis of directed triple systems by refinement, *Annals of Discrete Math.* **15** (1982), 97–103.

[5] J. Doyen, A note on reverse Steiner triple systems, *Discrete Math.* **1** (1972), 315–319.

[6] S. H. Y. Hung and N. S. Mendelsohn, Directed triple systems, *J. Comb. Th. Ser. A* **14** (1973), 310–318.

[7] A. Rosa, On reverse Steiner triple systems, *Discrete Math.* **1** (1972), 61–71.

[8] J. Seberry and D. Skillicorn, All directed BIBDs with $k = 3$ exist, *J. Comb. Th. Ser. A* **29** (1980), 244–248.

[9] L. Teirlinck, The existence of reverse Steiner triple systems, *Discrete Math.* **6** (1973), 301–302.

[10] L. Teirlinck, A simplification of the proof of the existence of reverse Steiner triple systems of order congruent to 1 modulo 24, *Discrete Math.* **13** (1975), 297–298.