Online Lab: Buoyancy

Name: Date:

Instructor: Section:

Theory

The **density** ρ of a substance of uniform composition is defined as the mass per unit volume:

$$\rho = \frac{m}{V} \tag{1}$$

where the units of ρ is kg/m^3 in the SI system and gm/cm^3 in the cgs system. Often, the density of material is given in terms of its value as compared to the density of fresh water (1000 kg/m^3). When the density is described in this manner, it is called the specific gravity of the material (e.g., lead has a specific gravity of 11.3 since it has a density of 11,300 kg/m^3).

The pressure P of a fluid is the ratio of the magnitude of the force exerted by the fluid to the surface area A that the fluid is pushing against:

$$P \equiv \frac{F}{A} \tag{2}$$

where the units of P is N/m^2= Pa (Pascal) in the SI system and dyne/cm^2 in the cgs system.

Let us now develop an equation that relates pressure to density:

- Consider a column of fluid of height h in a cylinder of cross-sectional area A as shown in the right-hand diagram of Figure 1. The force on the fluid is just the gravitational force

  $$F_g \;=\; mg \;=\; (\rho V)g \;=\; \rho(Ah)g,$$

  where we have used Eq. (1) and the volume formula for a cylinder ($V = Ah$).
- Using this force in Eq. (2) we get

  $$P = \frac{F_g}{A}\;\frac{\rho A h g}{A} = \rho g h \tag{3}$$

- This formula is valid for all states of matter (i.e., solids, liquids, gas) as shown in Figure 1.

If we wish to compare points between two different depths in a fluid, all we need to do is make use of Eq. (3) and subtract the pressure at the these two different depths (either h or y are acceptable here to describe depth):

$$\Delta P \;=\; \rho g \Delta h \;=\; \rho g \Delta y \tag{4}$$

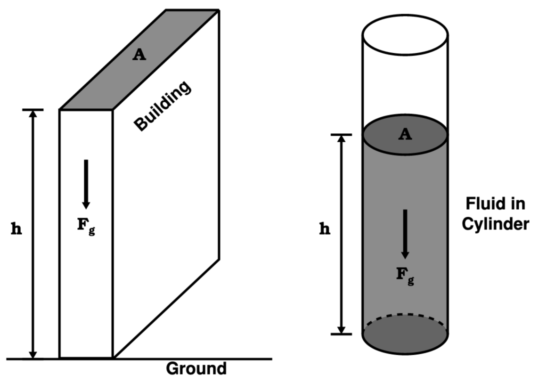


Figure 1: Examples of the relationship between density, pressure, and gravity

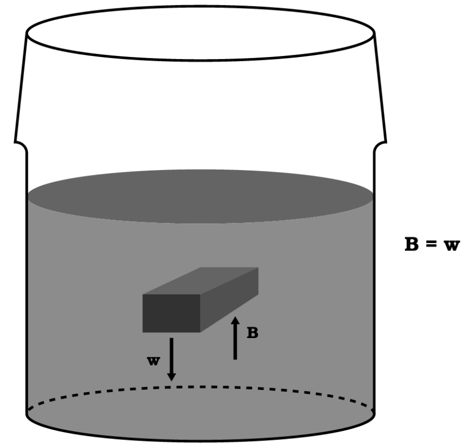


Figure 2: Archimedes' Principle demonstration with an object submerged in a fluid. Here the object remains stationary in the fluid since the buoyant force pointing upward is balanced by the weight of the object pointing downward.

Laws of Physics Involving Fluids

Pascal's Principle

Pascal's Principle states: Pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and to the walls of the containing vessel. Since we often measure pressure in fluids that are open to air at one end of a container, we can use Pascal's Principle to write

$$P = P_o + \rho g h \tag{5}$$

where P_o is the atmospheric pressure at the Earth's surface:

$$P_o = 1.01325 \times 10^5 \ Pa = 1.01325 \times 10^6 \ dynes/cm^2. \tag{6}$$

Note the pressure as described by Eqs. (3), (4), and (5) is not affected by the shape of the vessel.

Archimedes' Principle and Buoyancy

Archimedes' Principle: *Any body completely or partially submerged in a fluid is buoyed up by a force (called the "buoyant force") equal to the weight of the fluid displaced by the body*. **Note that the buoyant force always acts in a direction opposite to that of gravity**.

Mathematically:

$$\begin{aligned} B &= \Delta P \Delta A = (\rho f g h)(A) \\ &= \rho f g (hA) \ = \ \rho f_g V_f \\ &= \ M_f g \ = \ w_f , \end{aligned} \tag{7}$$

where B is the buoyant force imparted on the object by the fluid, ρ_f is the density of the fluid (note that the "f" subscript means "fluid"), A is the area of the object that the buoyancy force is pushing against, and w_f, V_f and M_f are the weight, volume and mass of the fluid displaced by the object. We can now ask the question, will an object sink or float when placed in a fluid? The weight of a submerged object is

$$w_o = m_o g = \rho_o V_o g,$$

where the "o" subscript represents "object." Then from Archimedes' Principle, for an object to float,

$$B \;=\; w_o \implies w_f \;=\; w_o$$

so

$$\rho_f g V_f \;=\; rho_o g V_o \tag{8}$$

or

$$\frac{\rho_o}{\rho_f} \;=\; \frac{V_f}{V_o} \tag{9}$$

If B $\neq w_o$ Then,

$$B - w_o \;=\; \rho_f g V_f - \rho_o g V_o \neq 0. \tag{10}$$

- If an object is completely submerged, $V_f = V_o$ and Eq. (10) becomes

$$B - w_o \;=\; (\rho_f - \rho_o)\, V_o g \tag{11}$$

- If $\rho_f > \rho_o$, B - $w_o > 0 \implies$ the object FLOATS!
- If $\rho_f < \rho_o$, B - $w_o < 0 \implies$ the object SINKS!

From Eq. (10), we see that there are two competing forces involved for submerged objects. From this, we can define an effective weight of such an object as

$$w_{eff} \;=\; w_o - B \;=\; F_g - F_b \;=\; mg - \rho_f V_f g. \tag{12}$$

Since $V_o = V_f$ and $V_o = m/\rho_o$, we can rewrite this equation as

$$w_{eff} \;=\; mg - \rho_f \left(\frac{m}{\rho_o}\right) g = mg\left(1 - \frac{\rho_f}{\rho_o}\right) = w_{air}\left(1 - \frac{\rho_f}{\rho_o}\right) \tag{13}$$

But what exactly do we mean by *effective weight*? Since weights are usually measured via its effect on an equilibrium force (*i.e.*, the normal force as the object rests on a scale, or the tension if hanging from a scale), the effective weight is nothing more than the tension on a cord that suspends the mass from a scale. As such, we also could write Eq. (12) as

$$T \;=\; w_o - B. \tag{14}$$

Note that if we were to take the object out of the water, we would get $T = w_o$, since the buoyancy force would then be zero

Effective Weight Example

An object weighing 300 N in air is immersed in water after being tied to a string connected to a scale. The scale now reads 265 N. Immersed in oil, the object appears to weigh 275 N. Find (a) the density of the object and (b) the density of the oil.

Solution (a):

This is nothing more than an effective weight type of problem. Note that we have $w_{air} = 300$ N, $w_{water} = 265$ N, and $w_{oil} = 275$ N, with w_{water} and w_{oil} representing our effective weights. We know that the density of water is $\rho_{water} = 1000$ kg/m^3. Using Eq. (13), we can solve for ρ_o, the density of the object:

$$w_{water} = w_{air}\left(1 - \frac{\rho_{water}}{\rho_o}\right)$$

$$\frac{w_{water}}{w_{air}} = 1 - \frac{\rho_{water}}{\rho_o}$$

$$\frac{\rho_{water}}{\rho_o} = 1 - \frac{w_{water}}{w_{air}}$$

$$\frac{\rho_{water}}{\rho_o} = 1 - \frac{265N}{300N} = 1 - 0.883 = 0.117$$

$$\frac{\rho_{water}}{\rho_o} = \frac{1}{0.117} = 8.57$$

$$\rho_o = 8.57\rho_{water} = 8.57\ (1000\ kg/m^3) = \boxed{8570\ kg/m^3}$$

Solution (b):

Now we use the same technique as in (a), but use the oil weight for the effective weight:

$$w_{oil} = w_{air}\left(1 - \frac{\rho_{oil}}{\rho_o}\right)$$

$$\frac{w_{oil}}{w_{air}} = 1 - \frac{\rho_{oil}}{\rho_o}$$

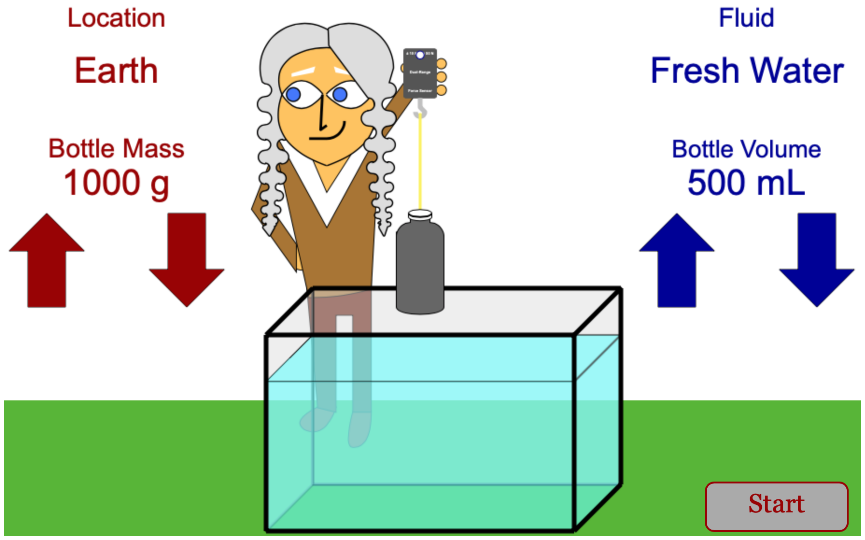
$$\frac{\rho_{oil}}{\rho_o} = 1 - \frac{w_{oil}}{w_{air}}$$

$$\frac{\rho_{oil}}{\rho_o} = 1 - \frac{275N}{300N} = 1 - 0.917 = 0.0833$$

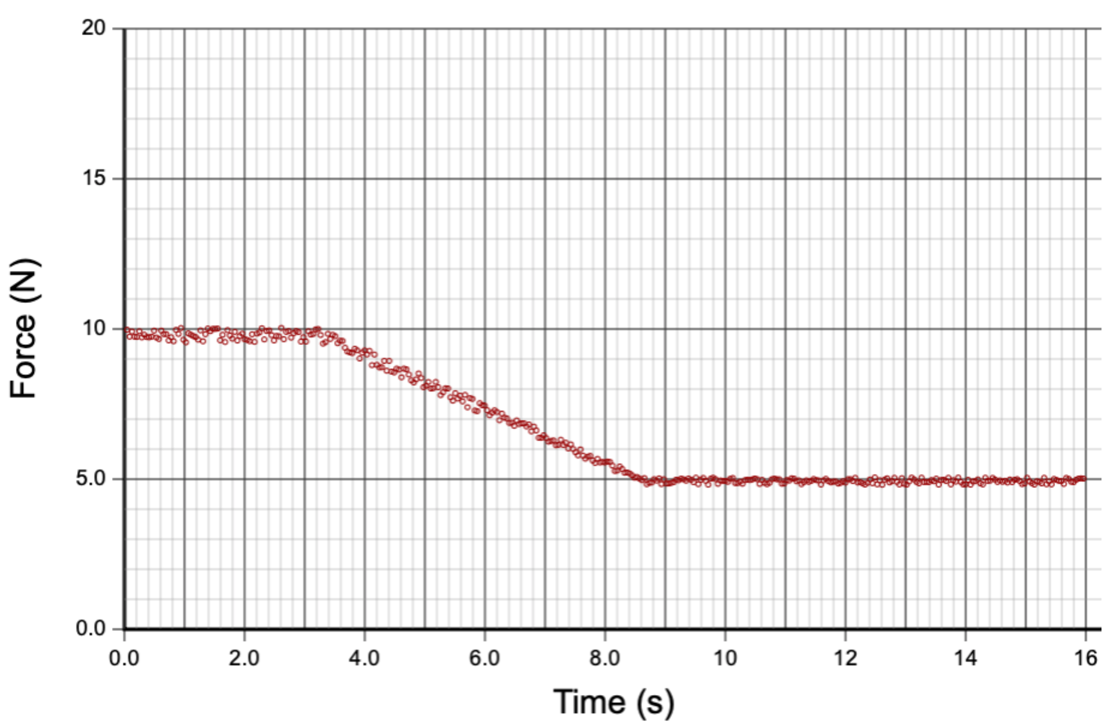
$$\rho_{oil} = 0.0833\rho_o = 0.0833\ (8570\ kg/m^3) = \boxed{714\ kg/m^3}$$

Online Buoyancy Experiment Setup Instructions.

1. Today's Online Lab we are going to use one of the physicsaviary.com simulations. Go to the following website:
   https://www.thephysicsaviary.com/Physics/Programs/Labs/ForceBuoyancy/
2. Click **Begin** on the bottom right of the simulation.
   The figure below shows what you should see on your screen



3. Obtain from your graph values for force in air, w_{air} and force when submerged in fluid, w_{fluid}. Subtract these forces to get buoyant force: $F_B = w_{air} - w_{fluid}$.



Part A: Buoyant Force vs Acceleration due to Gravity

1. **Set** the Bottle Volume to 500 mL and the Bottle Mass to 1000 grams.
2. **Set** the location of the experiment to Earth. The acceleration due to gravity, g at each location is given in Column 2.
3. Click Start and wait about 16 seconds for the bottle to be completely submerged.
4. After the bottle is completely submerged, scroll down to see the graph of your results.
5. Read and Record the weight of the bottle in air, w_{air} and the apparent weight of the bottle in water, w_{air} in Columns 3 and 4 below.
6. Repeat 2 - 5 for each location given in Column 1 of Table 1.
7. Calculate the Buoyant Force, F_B for each row and Record this value in Column 5.

Table 1: Data Results - Buoyant Force vs Acceleration due to Gravity

Location	g **(m/s/s)**	w_{air} **(N)**	w_{fluid} **(N)**	F_B **(N)**
Earth	9.81			
Moon	1.62			
Mars	3.71			
Venus	8.87			
Jupiter	24.79			
Vesta	0.22			

Data Analysis

1. **Use your results from Table 1 to create a scatter graph of F_B on the Y-axis versus g on the X-axis. Upload it with this Lab Report in D2L.**
   Measure and Record the slope of the line. Comment on the graph.

Part B: Buoyant Force vs Fluid Density

1. **Set** the Bottle Volume to 100 mL and the Bottle Mass to 2000 grams.
2. **Set** the location of the experiment back to Earth.
3. **Set** the fluid to Gasoline.
4. Click Start and wait about 16 seconds for the bottle to be completely submerged.
5. After the bottle is completely submerged, scroll down to see the graph of your results.
6. Read and Record the weight of the bottle in air, w_{air} and the apparent weight of the bottle in water, w_{air} in Columns 3 and 4 below.
7. Repeat 3 - 6 for each fluid given in Column 1 of Table 2.
8. Calculate the Buoyant Force, F_B for each row and Record this value in Column 5.

Table 2: Data Results - Buoyant Force vs Fluid Density

Fluid	**Density,** ρ (kg/m^3)	w_{air} **(N)**	w_{fluid} **(N)**	F_B **(N)**
Gasoline	737			
Crude Oil	827			
Fresh Water	1000			
Maple Syrup	1330			
Mercury	13,500			

Data Analysis

1. **Use your results from Table 2 to create a scatter graph of F_B on the Y-axis versus ρ on the X-axis. Upload it with this Lab Report in D2L.**
   Measure and Record the slope of the line. Comment on the graph.

Part C: Buoyant Force vs Bottle Volume

1. **Set** the fluid to Fresh Water and the Bottle Mass to 1000 grams.
2. **Select** the location of the experiment to Earth.
3. **Set** the Bottle Volume to 100 mL.
4. Click Start and wait about 16 seconds for the bottle to be completely submerged.
5. After the bottle is completely submerged, scroll down to see the graph of your results.
6. Read and Record the weight of the bottle in air, w_{air} and the apparent weight of the bottle in water, w_{air} in Columns 3 and 4 below.
7. Repeat 3 - 6 for four additional volumes and Record each Volume in Column 1 of Table 3.
8. Calculate the Buoyant Force, F_B for each row and Record this value in Column 4.

Table 3: Data Results - Buoyant Force vs Bottle Volume

Bottle Volume, V **(**mL**)**	w_{air} **(N)**	w_{fluid} **(N)**	F_B **(N)**
100			

Data Analysis

1. **Use your results from Table 3 to create a scatter graph of** F_B **on the Y-axis versus** V **on the X-axis. Upload it with this Lab Report in D2L.**
   Measure and Record the slope of the line. Comment on the graph.