General Physics Labs I (PHYS-2011)
EXPERIMENT FLUID–1:
Buoyancy

1 Introduction

Todays experiment will involve measuring the masses of various metals in both air and in water using a beam balance. The water will be contained in a Corning 400 milliliter (mL) plastic beaker (note that 1 mL = 1 cm^3). The volume of the metals will be measured by submerging them in water in a 50 milliliter glass graduated cylinder. From these measurements, you will determine the values of the density of each metal and the buoyant force supplied by the water on each. If one is supplied, you will also determine the density of an unknown fluid.

Table 1: Items Used in the Buoyancy Lab

PASCO Specific Heat (Mass) Set – SE-6849	Ohaus Beam Balance
400 milliliter plastic beaker	Physics string
50 milliliter glass graduated cylinder	

1.1 Density and Pressure

The (mass) **density** ρ of a substance of uniform composition is defined as the mass per unit volume:

$$\boxed{\rho = \frac{m}{V}\ ,} \tag{1}$$

where the units of ρ is kg/m^3 in the SI system and gm/cm^3 in the cgs system. Often, the density of material is given in terms of its value as compared to the density of fresh water (1000 kg/m^3). When the density is described in this manner, it is called the **specific gravity** of the material (*e.g.*, lead has a specific gravity of 11.3 since it has a density of 11,300 kg/m^3).

The **pressure** P of a fluid is the ratio of the magnitude of the force exerted by the fluid to the surface area A that the fluid is pushing against:

$$\boxed{P \equiv \frac{F}{A}\ ,} \tag{2}$$

where the units of P is N/m^2 = Pa (Pascal) in the SI system and dyne/cm^2 in the cgs system.

Let us now develop an equation that relates pressure to density.

- Consider a column of fluid of height h in a cylinder of cross-sectional area A as shown in the right-hand diagram of Figure 1. The force on th fluid is just the gravitational force
$$F_g = m\,g = (\rho\,V)\,g = \rho\,(A\,h)\,g\ ,$$
where we have used Eq. (1) and the volume formula for a cylinder ($V = A\,h$).

- Using this force in Eq. (2) we get
$$\boxed{P = \frac{F_g}{A} = \frac{\rho\,A\,h\,g}{A} = \rho\,g\,h\ .} \tag{3}$$

- This formula is valid for all states of matter (*i.e.*, solids, liquids, and gas) as shown in Figure 1.

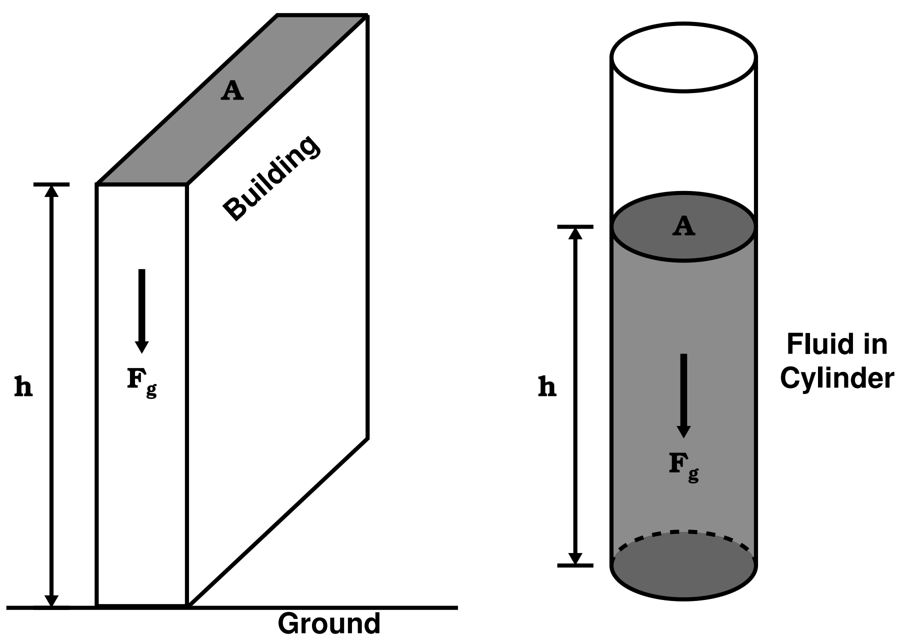

Figure 1: Examples of the relationship between density, pressure, and gravity.

If we wish to compare points between two different depths in a fluid, all we need to do is make use of Eq. (3) and subtract the pressure at the these two different depths (either h or y are acceptable here to describe depth):
$$\Delta P = \rho\,g\,\Delta h = \rho\,g\,\Delta y\ . \tag{4}$$

1.2 Laws of Physics Involving Fluids

1.2.1 Pascal's Principle

In words, **Pascal's Principle** states: *Pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and to the walls of the containing vessel.* Since we often measure pressure in fluids that are open to air at one end of a container, we can use Pascal's Principle to write

$$\boxed{P = P_\circ + \rho\, g\, h\ ,} \tag{5}$$

where $P_\circ$ is the atmospheric pressure at the Earth's surface:

$$\boxed{P_\circ = 1.01325 \times 10^5\ \text{Pa}\ = 1.01325 \times 10^6\ \text{dynes/cm}^2\ .} \tag{6}$$

Note that the pressure as described by Eqs. (3), (4), and (5) is not affected by the shape of the vessel.

1.2.2 Archimedes' Principle and Buoyancy

Archimedes' Principle: *Any body completely or partially submerged in a fluid is buoyed up by a force (called the "**buoyant force**") equal to the weight of the fluid displaced by the body.* **Note that the buoyant force always acts in a direction opposite to that of gravity.**

Mathematically:

$$\begin{aligned} B &= \Delta P \cdot A = (\rho_\mathrm{f}\, g\, h)(A) \\ &= \rho_\mathrm{f}\, g\, (h\, A) = \rho_f\, g\, V_\mathrm{f} \\ &= M_\mathrm{f}\, g = w_\mathrm{f}\,, \end{aligned} \tag{7}$$

where B is the buoyant force imparted on the object by the fluid, ρ_f is the density of the fluid (note that the "f" subscript means "fluid"), A is the area of the object that the buoyancy force is pushing against, and w_f, V_f and M_f are the weight, volume and mass of the fluid displaced by the object.

We can now ask the question, will an object sink or float when placed in a fluid? The weight of a submerged object is

$$w_\mathrm{o} = m_\mathrm{o}\, g = \rho_\mathrm{o}\, V_\mathrm{o}\, g\ ,$$

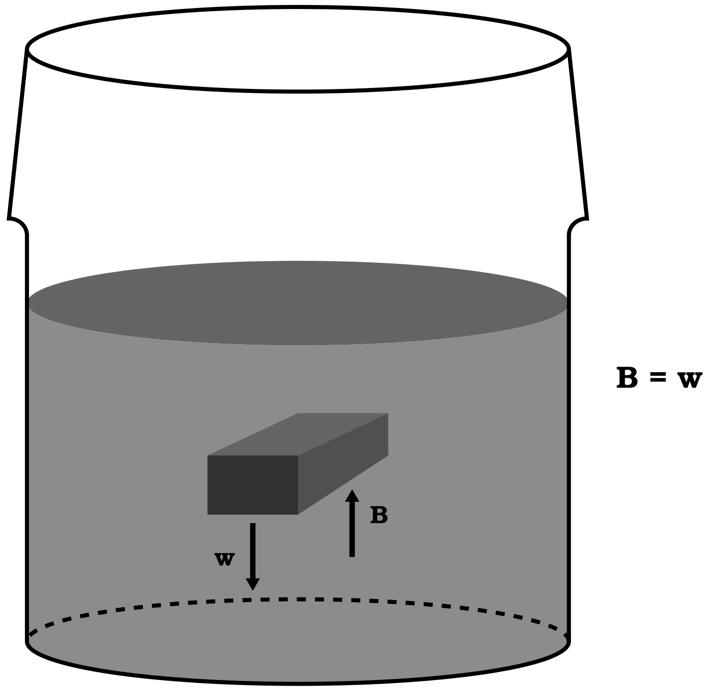

Figure 2: Archimedes' Principle demonstration with an object submerged in a fluid. Here the object remains stationary in the fluid since the buoyant force pointing upward is balanced by the weight of the object pointing downward.

where the "o" subscript represents "object." Then from Archimedes' Principle, for an object to float,

$$B = w_{\rm o} \quad \Longrightarrow \quad w_{\rm f} = w_{\rm o} \ ,$$

so

$$\rho_{\rm f}\, g\, V_{\rm f} = \rho_{\rm o}\, g\, V_{\rm o} \tag{8}$$

or

$$\boxed{\frac{\rho_{\rm o}}{\rho_{\rm f}} = \frac{V_{\rm f}}{V_{\rm o}} \ .} \tag{9}$$

What if $B \neq w_{\rm o}$? Then,

$$B - w_{\rm o} = \rho_{\rm f}\, g\, V_{\rm f} - \rho_{\rm o}\, g\, V_{\rm o} \neq 0 \ . \tag{10}$$

- If an object is completely submerged, $V_{\rm f} = V_{\rm o}$ and Eq. (10) becomes

$$B - w_{\rm o} = (\rho_{\rm f} - \rho_{\rm o})\, V_{\rm o}\, g \ . \tag{11}$$

- If $\rho_{\rm f} > \rho_{\rm o}$, $B - w_{\rm o} > 0 \Longrightarrow$ the object floats!

- If $\rho_{\rm f} < \rho_{\rm o}$, $B - w_{\rm o} < 0 \Longrightarrow$ the object sinks!

From Eq. (10), we see that there are two competing forces involved for submerged objects. From this, we can define an *effective weight* of such an object as

$$w_{\rm eff} = w_{\rm o} - B = F_{\rm g} - F_{\rm b} = m\,g - \rho_{\rm f}\,V_{\rm f}\,g \ . \tag{12}$$

Since $V_{\rm o} = V_{\rm f}$ and $V_{\rm o} = m/\rho_{\rm o}$, we can rewrite this equation as

$$\begin{aligned} w_{\rm eff} &= m\,g - \rho_{\rm f}\left(\frac{m}{\rho_{\rm o}}\right) g = m\,g\left(1 - \frac{\rho_{\rm f}}{\rho_{\rm o}}\right) \\ &= w_{\rm air}\left(1 - \frac{\rho_{\rm f}}{\rho_{\rm o}}\right) \ . \end{aligned} \tag{13}$$

But what exactly do we mean by *effective weight*? Since weights are usually measured via its effect on an equilibrium force (*i.e.*, the normal force as the object rests on a scale, or the tension if hanging from a scale), the effective weight is nothing more than the tension on a cord that suspends the mass from a scale. As such, we also could write Eq. (12) as

$$T = w_{\rm o} - B \ . \tag{14}$$

Note that if we were to take the object out of the water, we would get $T = w_{\rm o}$, since the buoyancy force would then be zero.

2 Effective Weight Example

An object weighing 300 N in air is immersed in water after being tied to a string connected to a scale. The scale now reads 265 N. Immersed in oil, the object appears to weigh 275 N. Find (a) the density of the object and (b) the density of the oil.

Solution (a):

This is nothing more than an effective weight type of problem. Note that we have $w_{\rm air} =$ 300 N, $w_{\rm water} = 265$ N, and $w_{\rm oil} = 275$ N, with $w_{\rm water}$ and $w_{\rm oil}$ representing our effective weights. We know that the density of water is $\rho_{\rm water} = 1000$ kg/m^3. Using Eq. (13), we can solve for $\rho_{\rm o}$, the density of the object:

$$\begin{aligned} w_{\rm water} &= w_{\rm air}\left(1 - \frac{\rho_{\rm water}}{\rho_{\rm o}}\right) \\ \frac{w_{\rm water}}{w_{\rm air}} &= 1 - \frac{\rho_{\rm water}}{\rho_{\rm o}} \end{aligned}$$

$$\begin{aligned}
\frac{\rho_{\rm water}}{\rho_{\rm o}} &= 1 - \frac{w_{\rm water}}{w_{\rm air}} \\
\frac{\rho_{\rm water}}{\rho_{\rm o}} &= 1 - \frac{265\ {\rm N}}{300\ {\rm N}} = 1 - 0.883 = 0.117 \\
\frac{\rho_{\rm o}}{\rho_{\rm water}} &= \frac{1}{0.117} = 8.57 \\
\rho_{\rm o} &= 8.57\rho_{\rm water} = 8.57\,(1000\ {\rm kg/m^3}) \\
&= \boxed{8570\ {\rm kg/m^3}\ .}
\end{aligned}$$

Solution (b):

Now we use the same technique as in (a), but use the oil weight for the effective weight:

$$\begin{aligned}
w_{\rm oil} &= w_{\rm air}\left(1 - \frac{\rho_{\rm oil}}{\rho_{\rm o}}\right) \\
\frac{w_{\rm oil}}{w_{\rm air}} &= 1 - \frac{\rho_{\rm oil}}{\rho_{\rm o}} \\
\frac{\rho_{\rm oil}}{\rho_{\rm o}} &= 1 - \frac{w_{\rm oil}}{w_{\rm air}} \\
\frac{\rho_{\rm oil}}{\rho_{\rm o}} &= 1 - \frac{275\ {\rm N}}{300\ {\rm N}} = 1 - 0.917 = 0.0833 \\
\rho_{\rm oil} &= 0.0833\rho_{\rm o} = 0.0833\,(8570\ {\rm kg/m^3}) \\
&= \boxed{714\ {\rm kg/m^3}\ .}
\end{aligned}$$

3 Procedure

In the example above, we carried out the calculations using the SI system of measuring units. In today's experiment, we will be using cgs units. Carry out each measurement described below <u>three</u> different times, <u>recording</u> and <u>averaging</u> these measurements. At this point, carry out the following procedural steps:

1. First, using four different metal objects labeled A, C, D, and E, we will measure the masses of each using the Ohaus Beam Balance. Each of these objects (see Figure 3 on the next page) have different densities. You will be determining the density of each and making comparisons to the densities of various metals listed in Table 2. Each mass has a small hole in one end allowing it to be attached to a short length of string with a loop at the other end.

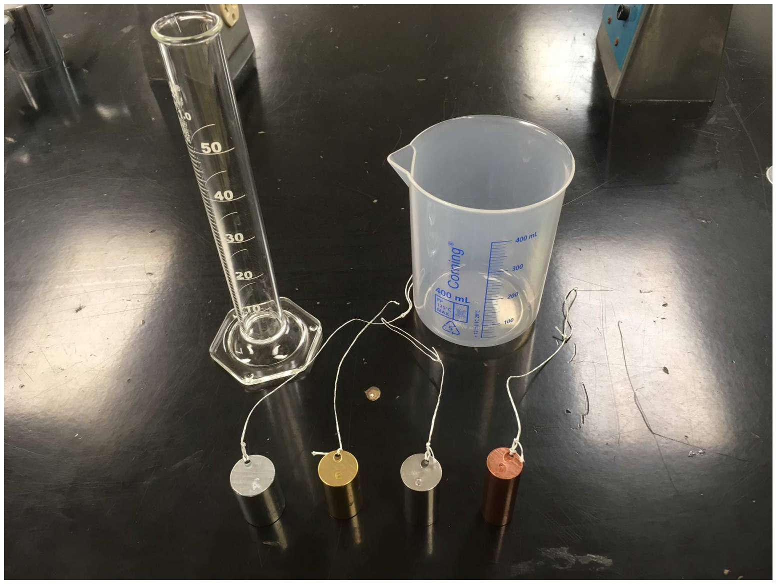

Figure 3: The mass set, graduated cylinder, and plastic beaker used in today's experiment.

Table 2: Densities of Various Metal

Metal	**Density (gm/cm^3)**	**Metal**	**Density (gm/cm^3)**
Aluminum	2.71	Mercury	13.56
Brass	8.60	Platinum	21.50
Copper	8.96	Stainless Steel	7.70
Lead	11.30	Zinc	7.14

2. Place the Ohaus Beam Balance on top of the metal rod clamped to the table. Zero the balance and then attach a mass to the balance placing the string loop on the small metal hook underneath the balance pan as shown in Figure 4 on the next page. Record the mass of the metal in grams as measured in "air".

3. Now fill the 400 ml plastic beaker with approximately 300 ml of water. Place the suspended mass in the water until it is completely submerged and record the measured mass of the metal now immersed in "water". (See Figure 5 on the next page.)

4. Repeat the above procedure for the other metal masses.

5. Now fill the graduated cylinder with approximately 30 mL of water. The exact level of water is not critical, but determining an accurate water level <u>is</u> critical. You must put your eye at the level of the meniscus to read the value as precisely as possible as shown in Figure 6.

Figure 4: Hooking the unknown masses to the Ohaus Beam Balance.

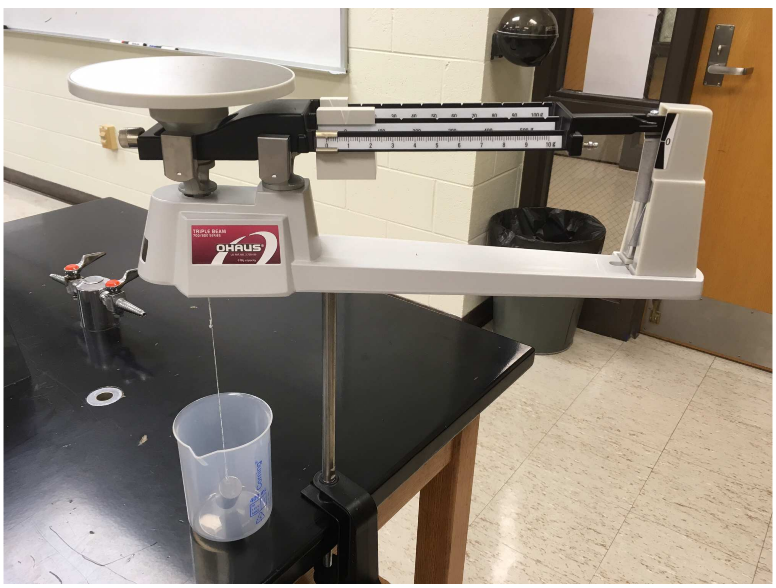

Figure 5: Placing the unknown masses in the fluid in the 400 mL plastic beaker.

Be certain there are no air bubbles in the water and record the water level in the cylinder in mL by determining the location of the meniscus. Do your best to determine this value to the nearest 0.5 mL. Now slowly lower one of the metal masses into the cylinder until it rests on the bottom. Again, making sure there are no air bubbles, record the water level with the mass submerged. Remove the mass from the water and repeat the above procedure two more times. You will use the average of these measurements for your calculations.

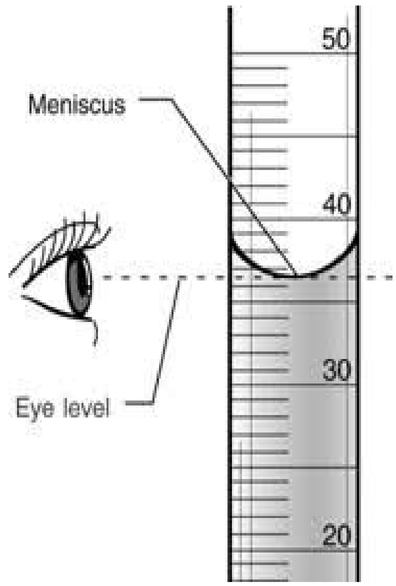

Figure 6: The proper technique for measuring the fluid level in the graduated cylinder.

6. Repeat the above procedure for the other metal masses.
7. Empty the plastic beaker and dry off all masses once your measurements are complete.
8. **Optional Measurements:** If an unknown fluid is provided, use a second 400 ml plastic beaker and fill it with approximately 250 ml of the fluid. A fifth metal (labeled B) will be used to determine the density of this unknown fluid. Metal B is aluminum. Following the same procedure as for the 4 unknown metals, suspend the aluminum mass from the Ohaus Beam Balance and measure its mass both in air and immersed in the fluid. Record both masses. Table 3 supplies densities for various liquids. (**Be certain to return the fluid to the container provided and to clean the beaker as directed by your lab instructor once the measurement is completed!**)

Table 3: Densities of Various Liquids

Liquid	**Density (gm/cm^3)**	**Liquid**	**Density (gm/cm^3)**
isopropyl alcohol	0.79	glycerin	1.26
ethylene glycol	1.10	turpentine	0.85
glucose	1.40	water	1.00

4 Analysis

Following the effective weight example [Solution (a)] given in these lab instructions, calculate the density of each unknown metal. Compare the densities you calculate to values listed in the table for metals. From this comparison, identify the material for each and then calculate the percent error for your determined densities. Address the uncertainties in your measurements.

A simpler approach to determining density would be to use Eq. (1) and your measured values for the volume of each mass. Using the average of the difference in water volume measurements (submerged mass versus initial water in the cylinder), determine the volume of each metal mass and then its density from Eq. (1). Compare these results with the previous density calculations for each metal and again address the uncertainties in the volume measurement.

Use Eq. (7) to determine the **buoyant force** imparted by the water to each metal mass. Finally, if an unknown fluid was supplied, following the effective weight example [Solution (b)] given in these lab instructions, calculate the density of the unknown fluid. Compare the density you calculate to values listed in the table for liquids. From this comparison, identify the liquid and then calculate the percent error for your determined density. Address the uncertainties in your measurements.