General Physics Labs I (PHYS-2011)

EXPERIMENT MEAS–4:
Graphs and Graphical Analysis

1 Introduction

In physics, graphs are often used to provide a visual presentation of data, which may be much more meaningful than just presenting numbers in a table. When data are presented in graphical form, the shape of the graph can vary considerably. Figure 1 shows some of the different shapes that you may encounter in any of the physical sciences. Some experiments in this course will require students to produce graphs during the lab itself.

![Graphs](image)

Sometimes data points are displayed in graphs by simply drawing straight lines connecting each point as shown in Figure 1(a). At other times, the data points suggest a specific trend, such as a linear relationship shown in Figure 1(b) or an exponential relationship shown in Figure 1(c). Perhaps the data shows no particular mathematical pattern at all. Figure 1(d) shows such plot where the data is represented by a bar chart.

*It is important to realize that experimental data will almost never fit any given shape exactly.* In Figure 1(b), note that the straight line has been “fitted” to the data points, and that some of the points (most of them, in fact) don’t lie exactly on the line. This is a normal circumstance, and just reflects the fact that experimental results are hardly ever perfectly precise.
2 Linear Relationships: The Straight-Line Graph

We now examine data that follow a linear relationship. When this is the case, a straight line can be fitted to the data following a “linear regression analysis” or a linear “least-squares fit” to the data. The equation that represents a straight line is

\[ y = y_0 + mx, \]

where \( x \) represents the independent variable and \( y \) represents the dependent variable. The ‘independent’ variable is typical plotted along the horizontal axis and the ‘dependent’ variable plotted along the vertical axis. In the equation above, \( y_0 \) is the \( y \)-intercept, the location where the “curve” (here a straight line) crosses the \( y \) axis at \( x = 0 \), and \( m \) is called the slope of the line. The ‘slope’ is a measure of the rate at which the \( y \)-value changes when the \( x \)-value changes. Note that typical independent variables used in physics are \( x \) (displacement in the horizontal direction), \( r \) (displacement in the radial direction), \( t \) (time), \( \theta \) or \( \phi \) (angles), and \( s \) (arclength). There are many different variables to represent dependent data. Some of the more common ones are \( y \) (displacement in the vertical direction), \( F \) (force), \( W \) (work), \( E \) (energy), \( p \) (linear momentum), and \( L \) (angular momentum).

To understand the mathematics of a straight line, please refer to Figure 2.

\[ \text{Figure 2: Straight Line } y = y_0 + mx. \]
To determine the y-intercept directly from the graph, we only have to read it off of the y-axis. To determine the slope, \( m \), we can construct a right-angle triangle, as shown by the dotted lines and the shaded area in Figure 2. The actual two points chosen on the straight line to determine the slope do not matter since all points on the straight line will have the same slope. Remember that these are points on this line. If the data points are not on this line, they should not be used. We can then calculate the slope with the equation:

\[
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.
\]

Thus we can find the slope \( m \) if we know, or if we can measure, \( \Delta y \) and \( \Delta x \) from any two points on the line.

Once we know \( m \) and the coordinates \((x_1, y_1)\) of any point on the line, then coordinates \((x, y)\) of any other point on the line can be found:

\[
m = \frac{y - y_1}{x - x_1}, \quad \text{and hence} \quad (y - y_1) = m(x - x_1), \quad \text{and finally} \quad y = y_1 + m(x - x_1).
\]

Now, if set \( x_1 = 0 \), then \( y_1 = y_o \implies \text{the y-intercept} \). The equation above then becomes:

\[
y = y_o + mx,
\]

the linear equation describing a straight line.

We need to note here that not all data follow a straight-line pattern or linear distribution. In the next section, we discuss the steps one should take when analyzing graphs.

### 3 Graphical Analysis

Let’s assume that we have carried out an experiment and tabulated the data obtained from the measurements made during this experiment. Here are the steps one should take when analyzing this data.

#### 3.1 Linear Data

1. From your data table, determine which columns of the data should be analyzed. Note that there may be more than just two columns. If so, then you may need to make multiple graphs of data pairs. Then make a two-dimensional graph of the data pairs, choosing one column to represent the independent data (such as horizontal position or time) and another column to represent the dependent data (such as vertical position).
2. Plot your data on graph paper locating the independent data on the horizontal \((i.e., \ x)\) axis and the dependent data on the vertical \((i.e., \ y)\) axis. Make sure you label the axes with what is being plotted along with the units of the data \((e.g., \ ‘x (meters)’\) on the vertical axis vs ‘t (seconds)’ on the horizontal axis.

3. Look to see if the data appears to follow a straight line (see Figure 1(b) as an example). If so, take a ruler and position it such that you get the least spread of data points away from the ‘ruled’ edge of the ruler. The draw a straight line along that ‘ruled’ edge.

4. If the data does not appear to follow a straight-line pattern \((e.g., \ see \ Figure \ 1(c))\), one can use the methods of Section 3.2 below to replot the data assuming a non-linear variation. For your initial graph, just leave the plotted data as unconnected data points.

If the data does appear to follow a straight-line pattern, then analyze the data with the following steps:

1. Pick two separate points on the drawn straight line and determine the horizontal and vertical positions of these points the best you can. Note that there will be an uncertainty to these measurements. Try to estimate the size of these uncertainties (typically the least count of the grid on the graph paper).

2. Calculate the slope of the line using Eq. (2).

3. Locate the position on the vertical axis where the drawn straight line crosses this axis – this is your \(y\)-intercept.

4. From these measurements, write out the linear equation that describes these data following the form of Eq. (1). \textbf{Note, as always, to pay attention to significant digits when writing this equation.}

Now looking at the units on the vertical and horizontal axes, does the slope, \(m\), have special meaning? For instance, if the vertical axis contains the position \((\text{in meters})\) of an object along a flat surface, and the horizontal axis contains the time \((\text{in seconds})\) that this object was at this position, then the slope will have units of meters/second which are units associated with velocity. Hence the slope of the line is proportional to the velocity of the object.

\subsection*{3.2 Non-Linear Data}

What if our plot does not display a linear relationship? Then we can try to determine if the data might follow a parabolic distribution by plotting the data as is on the vertical axis
(say \( y \), height above the ground), and the square of the data on the horizontal axis (say \( t^2 \), time-squared when the object was at that height). If we then see a linear relationship from this graph, then we know that \( y \) is a function of \( t^2 \) and the slope of this line has units \( \text{m/s}^2 \), which are the units of acceleration. Hence the slope is proportional to the acceleration. Be careful, though. The constant of proportionality is not always ‘1’ (one).

There are a variety of different types of curves that might be encountered in physics. Figures 1(b) and 1(c) show a linear distribution of data and an exponential decay of data. Figure 3 displays some of the other more common types of curves that one may encounter.

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![Graphs and Graphical Analysis](image.png)

**Figure 3: Common Curves Seen in Physics**

One often encounters parabolic distributions of data in physics (see Figure 3(a)), for instance when plotting the path of a projectile in a gravitational field. A hyperbolic distribution of data points is seen when plotting the pressure of a confined gas as a function of
volume as shown in Figure 3(b). When measuring the decibel level of sound as a function of the sound’s intensity, a common logarithm distribution of data is recorded as shown in Figure 3(c). Finally when plotting some sort of oscillatory motion, the measurements follow a sinusoidal pattern (see Figure 3(d)). In each case, one can replot the data in a functional form [for example, $P$ (pressure) vs. $1/V$ (reciprocal volume), or $\log \beta$ vs. $\log I$ (sound intensity)] to gain insight into the mathematical dependence of one variable on another.

4 Procedure

This is a “paper experiment,” you will be provided data to analyze according to methods described in the previous sections. Today’s experiment will give you practice analyzing data through graphical analysis. This is important since you will be making use of these techniques for the remainder of this course and in the General Physics II Laboratory course. Below are exercises to carry out as practice for the data sheets you will be given in class. Feel free to make use of the ‘graph paper’ available at a link on the course web page.

Practice Exercises in Graphical Analysis

1. An object on a frictionless air track is measured to have the following instantaneous velocities at various times as recorded in the table below:

<table>
<thead>
<tr>
<th>time $t$ (sec)</th>
<th>1.00</th>
<th>1.50</th>
<th>2.00</th>
<th>2.50</th>
<th>3.00</th>
<th>3.50</th>
<th>4.00</th>
<th>4.50</th>
<th>5.00</th>
<th>5.50</th>
<th>6.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>velocity $v$ (m/sec)</td>
<td>30.6</td>
<td>25.4</td>
<td>20.7</td>
<td>15.4</td>
<td>10.6</td>
<td>5.8</td>
<td>0.7</td>
<td>-3.9</td>
<td>-8.8</td>
<td>-13.8</td>
<td>-18.7</td>
</tr>
</tbody>
</table>

(a) Plot this data on graph paper with $t$ on the horizontal axis and $v$ on the vertical axis. Use a “reasonable” scale for your axes (e.g., 1 inch = 1 sec for the horizontal axis). Be sure you label the graph and the axes properly including units.

(b) Decide whether or not a straight-line “curve” is appropriate, and if so, fit a straight line as best you can.

(c) From your graph, find $v_0$ (i.e., the $v$-intercept) and $m$ (i.e., the slope), where $v_0$ is the velocity at $t = 0$. To do this, you might need to extend your straight line somewhat to the left. This is called extrapolation, where we assume that the behavior prior to $t = 1.0$ sec is similar to the behavior after $t = 1.0$ sec. (Please note that such an assumption is not always justified.)

(d) What physical quantity does the slope $m$ correspond to? Give a reason for your answer.
2. The graph below describes the change in temperature along a steel rod as heat is conducted through this rod.

![Graph of Temperature vs. Position on Rod](image)

(a) Determine the temperature $T_1$ of the rod at position $x_1 = 8.0$ cm, and the temperature $T_2$ at position $x_2 = 22.0$ cm. (Read these values directly from the graph.) Use these to determine the slope of this line (including units):

$$m = \frac{\Delta T}{\Delta x} = \frac{T_2 - T_1}{x_2 - x_1}.$$

(b) Heat is being applied to the left end of the rod located at position $x = 0$ (zero) on the above graph. Use the value of $m$ determined in (a) above to calculate $T_o$, the temperature at the left end of the rod. Make use of the equation below to help you determine this value:

$$T = T_o + m x.$$

(c) Next, extrapolate the straight-line curve on the graph above to find $T_o$ \(i.e.,\) the value of $T$ when $x = x_o = 0$) by simply reading it off of the vertical axis. Compare this result with the value of $T_o$ determined in part (b).
3. Below is a table of the position of a moving object versus time:

<table>
<thead>
<tr>
<th>time t (sec)</th>
<th>0.50</th>
<th>1.00</th>
<th>3.00</th>
<th>5.00</th>
<th>6.00</th>
<th>7.00</th>
<th>8.00</th>
<th>9.00</th>
<th>10.00</th>
<th>11.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>position s (meters)</td>
<td>12.0</td>
<td>17.0</td>
<td>38.0</td>
<td>133</td>
<td>189</td>
<td>250</td>
<td>325</td>
<td>410</td>
<td>503</td>
<td>606</td>
</tr>
<tr>
<td>time^2 t^2 (sec^2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Graph the position $s$ versus time $t$ placing time on the horizontal axis. Is a straight line a good fit to the data?

(b) Now graph the position $s$ versus time squared $t^2$ on a separate plot. You will first need to fill in the $t^2$ row in the data table above. What conclusions can be drawn from this graph?

(c) From the graph in part (b), determine the initial position $s_o$ and the acceleration $a$ in the equation:

$$s = s_o + \frac{1}{2}a t^2.$$ 

Assume that this equation describes the motion of this object. Describe the steps you took to determine these values. Also, does the numerical value of the slope correspond directly to ‘$a$’, why or why not? (Note that you may need to draw a separate graph with an expanded $t^2$ scale near the origin to more accurately determine $s_o$.)