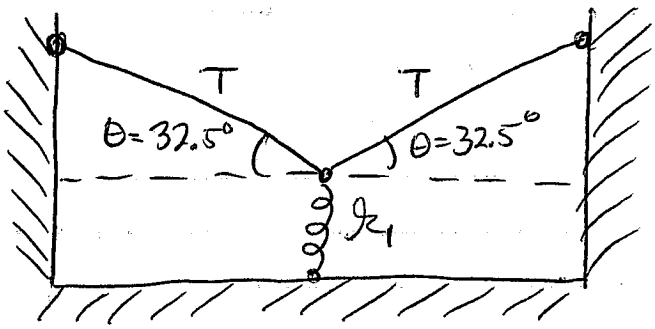
PHYS-2020: GEN. PHYSICS II

SUPPLEMENTAL PROBLEM SET 3

SOLUTIONS

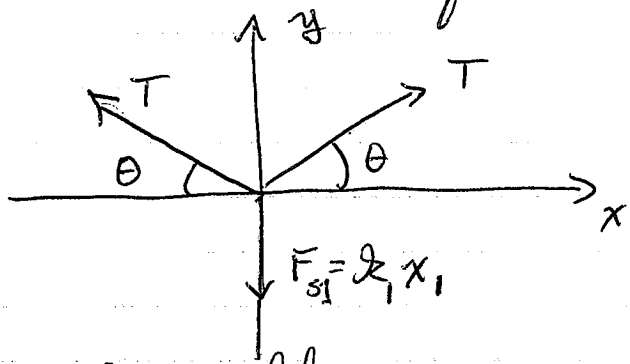
1.



$k_1 = 5.60 \times 10^4$ N/m

$T = 210$ N

a) We first need to draw a free-body diagram:



Since we are in equilibrium:

$$\sum F_y = T\sin\theta + T\sin\theta - k_1 x_1 = 0$$

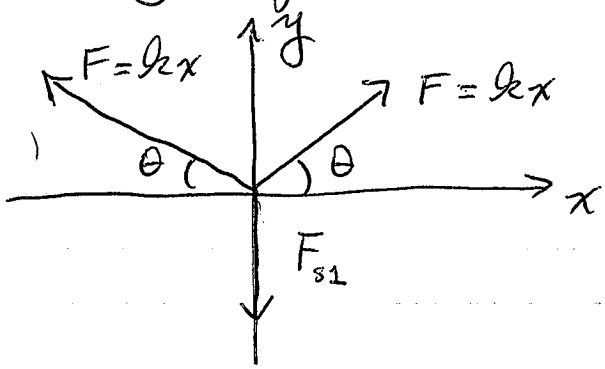
$$2T\sin\theta = k_1 x_1$$

$$x_1 = \frac{2T\sin\theta}{k_1} = \frac{2(210\,N)\sin 32.5^\circ}{5.60\times10^4\,N/m}$$

$$= 4.03\times10^{-3}\,m = \boxed{4.03\ mm}$$

b) The ropes are now replaced with springs of spring constant k. These springs are stretched $x = 2x_1$ from their equilibrium positions

The free-body diagram is



$$\sum F_y = kx\sin\theta + kx\sin\theta - k_1 x_1 = 0$$

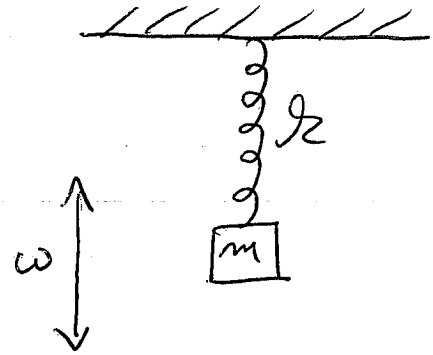
$$2kx\sin\theta = k_1 x_1$$

$$k = \frac{k_1 x_1}{2x\sin\theta} = \frac{k_1 x_1}{2(2x_1)\sin\theta}$$

$$= \frac{k_1}{4\sin\theta} = \frac{5.60\times10^4\ N/m}{4\sin 32.5^\circ}$$

$$= \boxed{2.61\times10^4\ N/m}$$

2.



$k = 16.0\ N/m$
$A = 20.0\ cm = 0.200\ m$
$v_{MAX} = 40.0\ cm/s = 0.400\ m/s$

Use conservation of energy here. Take the equilibrium position, $x=0$, as our initial position and the maximum extension position as our final position. Note when $x_i = 0$, $v_i = v_{MAX}$ and when $x_f = A$, $v_f = 0$. Let's assume that the change of height is small so that

$$(PE_g)_i \approx (PE_g)_f$$

$$(KE + PE_s + PE_g)_i = (KE + PE_s + PE_g)_f$$

$$(KE + PE_s)_i = (KE + PE_s)_f$$

$$\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2$$

$$\frac{1}{2}mv_{MAX}^2 = \frac{1}{2}kA^2$$

$$m = \frac{kA^2}{v_{max}^2} = \frac{(16.0\ N/m)(0.200\ m)^2}{(0.400\ m/s)^2} = 4.00\ kg$$

$$w = F_g = mg = (4.00\ kg)(9.80\ \tfrac{m}{s^2}) = \boxed{39.2\ N}$$

3.

$L_T = 0.9927\ m$ (Tokyo), $L_c = 0.9942\ m$ (Camb.)

$$T_T = 2\pi\sqrt{\frac{L_T}{g_T}} \quad \text{and} \quad T_c = 2\pi\sqrt{\frac{L_c}{g_c}}$$

Since a "seconds" pendulum travels through the equilibrium position every second, the total period is 2 times that time, or

$$T_T = T_c = 2.00\ s$$

Let's write the period equations as a ratio:

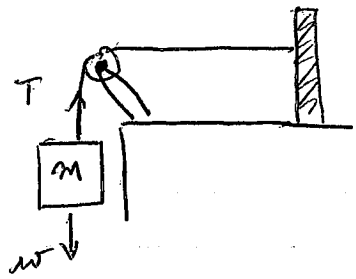
$$\frac{T_T}{T_c} = 1 = \frac{2\pi\sqrt{L_T/g_T}}{2\pi\sqrt{L_c/g_c}} = \frac{\sqrt{L_T/g_T}}{\sqrt{L_c/g_c}}$$

$$\frac{L_T/g_T}{L_c/g_c} = 1, \text{ or } \frac{L_T}{L_c}\frac{g_c}{g_T} = 1$$

$$\frac{g_c}{g_T} = \frac{L_c}{L_T} = \frac{0.9942\ m}{0.9927\ m} = \boxed{1.0015}$$

The surface gravity is 0.15% larger in Cambridge as compared to Tokyo.

4.



$m = 3.0\ kg$, $v = 24\ m/s$

Mass is not moving $\Rightarrow$ static equilibrium

a) What is μ?

$$\sum F_y = T - w = T - mg = 0$$

$$T = mg = (3.0\ kg)(9.80\ m/s^2) = 29\ N$$

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{T}{\mu}}$$

$$\mu = \frac{T}{v^2} = \frac{29\ N}{(24\ m/s)^2} = \boxed{0.051\ kg/m}$$

b) When $m = 2.00\ kg$, the tension is

$$T = mg = (2.0\ kg)(9.80\ \tfrac{m}{s^2}) = 20\ N$$

The speed of the transverse wave is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{20\ N}{0.051\ kg/m}} = \boxed{20\ m/s}$$

5.

$f = 700\ Hz$, $\lambda = 0.50\ m$, $T = ?$ (note $Hz = \frac{1}{s}$)

The speed of the sound wave is

$$v = f\lambda = (700\ Hz)(0.50\ m) = 3.5 \times 10^2\ m/s$$

Now use $v = (331\ \frac{m}{s})\sqrt{\frac{T}{273\ K}}$ and solve for T:

$$\sqrt{\frac{T}{273K}} = \frac{v}{331\ m/s}, \quad \frac{T}{273K} = \left(\frac{v}{331\ m/s}\right)^2$$

$$T = (273\ K)\left(\frac{v}{331\ m/s}\right)^2 = (273\ K)\left(\frac{350\ m/s}{331\ m/s}\right)^2$$

$$= 305\ K$$

$$T_c = T - 273 = 305 - 273 = \boxed{32^\circ C}$$

6.

$\beta_{ORC} = 85\ dB$, $\beta_V = 70\ dB$, where ORC $\equiv$ full orchestra, V $\equiv$ violin. Calculate I_{ORC}/I_V (I = intensity).

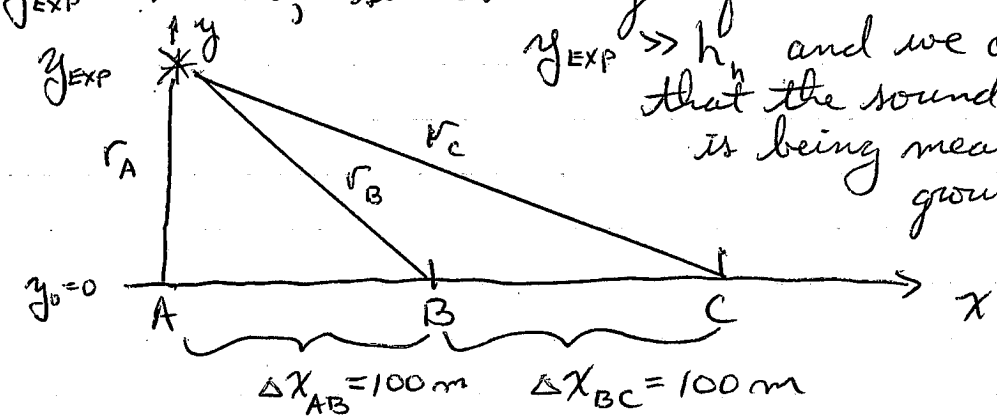
Note that $\beta = 10 \log(I/I_0)$, where $I_0 = 10^{-12}\ \frac{W}{m^2}$. So taking the antilog and setting up a ratio, we get $I = I_0\, 10^{\beta/10}$

$$\frac{I_{ORC}}{I_V} = \frac{I_0\, 10^{\beta_{ORC}/10}}{I_0\, 10^{\beta_V/10}} = 10^{(\beta_{ORC} - \beta_V)/10} = 10^{(85-70)/10}$$

$$= 10^{1.5} = 32 \text{ or } \boxed{I_{ORC} = 32\, I_V}$$

7.

$y_{EXP} = 100\ m$, since the height of humans is $h_h < 2\ m$, $y_{EXP} \gg h_h$ and we can assume that the sound wave is being measured at ground level.



The path length that the sound travels to each observer is

$$r_A = \Delta y = y_{EXP} = 100\ m$$

$$r_B = \sqrt{\Delta x_{AB}^2 + \Delta y^2} = \sqrt{(100\,m)^2 + (100\,m)^2}$$

$$= 141\ m$$

$$r_C = \sqrt{(\Delta x_{AB} + \Delta x_{BC})^2 + \Delta y^2}$$

$$= \sqrt{(100\,m + 100\,m)^2 + (100\,m)^2} = 224\ m$$

The intensity at distance r from the source is

$$I = \frac{\mathcal{P}}{4\pi r^2} = \frac{(\mathcal{P}/4\pi)}{r^2},$$ where $\mathcal{P}$ = power of the explosion which is constant for all 3 observers.

a) $$\frac{I_A}{I_B} = \frac{(\mathcal{P}/4\pi)/r_A^2}{(\mathcal{P}/4\pi)/r_B^2} = \left(\frac{r_B}{r_A}\right)^2$$

$$= \left(\frac{141\,m}{100\,m}\right)^2 = \boxed{2.0}$$

b) $$\frac{I_A}{I_C} = \frac{(\mathcal{P}/4\pi)/r_A^2}{(\mathcal{P}/4\pi)/r_C^2} = \left(\frac{r_C}{r_A}\right)^2$$

$$= \left(\frac{224\,m}{100\,m}\right)^2 = \boxed{5.0}$$

8.

Since the observer hears a reduced frequency, the source and the observer are getting farther apart. Hence, the bicyclist is $\boxed{\text{behind the car}}$.

With the bicyclist (observer) behind the car (source) and both moving in the same direction, the observer moves <u>towards</u> the source ($v_o > 0$) while the source moves <u>away</u> from the observer ($v_s < 0$). Thus

$f_o' = 415\,Hz$

$v_o = +v_{bike} = +\frac{v_{CAR}}{3}$ and $v_s = -v_{CAR}$, $f_s = 440\,Hz$

Obs. freq: $$f_o' = f_s\left(\frac{v+v_o}{v-v_s}\right) = f_s\left[\frac{v+v_{CAR}/3}{v-(-v_{CAR})}\right]$$

$$= f_s\left(\frac{v+v_{CAR}/3}{v+v_{CAR}}\right),$$ where v = speed of sound

Solving for v_{CAR} gives

$$f_o'(v + v_{CAR}) = f_s\left(v + \frac{v_{CAR}}{3}\right)$$

$$f_o' v_{CAR} - \frac{1}{3} f_s v_{CAR} = f_s v - f_o' v$$

$$v_{CAR}\left(f_o' - \frac{1}{3} f_s\right) = (f_s - f_o') v$$

$$v_{CAR} = \frac{f_s - f_o'}{f_o' - \frac{1}{3} f_s} v$$

v here is the velocity of sound, since they did not give us a temperature of the air, we will use room temperature of 293 K and use $v = 343$ m/s (see page 429 of the textbook), so

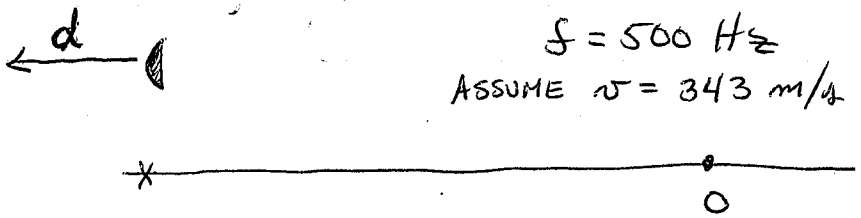
$$v_{CAR} = \frac{440\text{ Hz} - 415\text{ Hz}}{415\text{ Hz} - \frac{1}{3}\cdot 440\text{ Hz}}(343\text{ m/s})$$

$$= \frac{25\text{ Hz}}{268.3\text{ Hz}}(343\text{ m/s}) = 0.093\left(343\ \frac{\text{m}}{\text{s}}\right)$$

$$= \boxed{32\text{ m/s}}$$

(Note that a statement was made at the beginning of the Problems in this chapter to use $v = 345$ m/s if the speed of sound is not given. However, for this and other problems from this chapter, I will use $T_{ROOM} = 293\text{ K} \neq 20°\text{C}$ with $v = 343$ m/s for the sound speed.)

9.



$f = 500\ Hz$

ASSUME $v = 343\ m/s$ for $T = 293\ K$

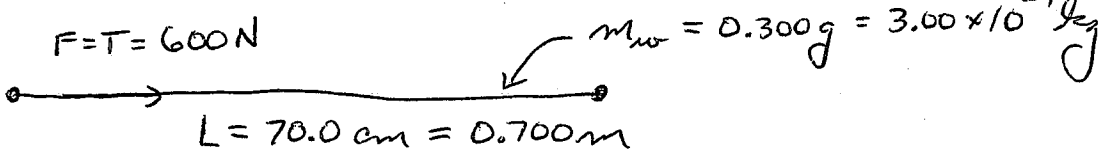
$$\lambda = \frac{v}{f} = \frac{343\ m/s}{500\ /s} = 0.686\ m$$

a) To produce destructive interference, the speaker should be moved back a distance

$$d = \frac{\lambda}{2} = \frac{0.686\ m}{2} = \boxed{0.343\ m}$$

b) If top speaker is moved back 2d in distance, the speakers will be separated by a full wavelength and $\boxed{\text{constructive interference}}$ will occur.

10.



Mass/length: $\mu = \frac{m_w}{L} = \frac{3.00 \times 10^{-4}\ kg}{0.700\ m} = 4.29 \times 10^{-4}\ kg/m$

Wave vel: $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{600\ N}{4.29 \times 10^{-4}\ kg/m}} = 1.18 \times 10^{3}\ m/s$

Fundamental: $\lambda_1 = 2L = 1.40\ m$, $f_1 = \frac{v}{\lambda_1} = \frac{1.18 \times 10^{3}\ m/s}{1.40\ m} = \boxed{845\ Hz}$

2nd harmonic: $f_2 = 2f_1 = \boxed{1690\ Hz}$, 3rd: $f_3 = 3f_1 = \boxed{2540\ Hz}$

11.

$L = 2.8\ cm = 2.8 \times 10^{-2}\ m, \quad v = 340\ m/s$

Hearing would be best at the fundamental resonance, so $\lambda = 4L = 4(2.8 \times 10^{-2}\ m)$
$= 0.112\ m$

$$f = \frac{v}{\lambda} = \frac{340\ m/s}{0.112\ m} = 3000\ Hz = \boxed{3.0\ kHz}$$

12.

$B_{MAX} = 1.5 \times 10^{-7}\ T, \quad v = c = 3.0 \times 10^{8}\ m/s$

a) $\frac{E_{MAX}}{B_{MAX}} = c, \quad E_{MAX} = cB_{MAX} = (3.0 \times 10^{8}\ \frac{m}{s})(1.5 \times 10^{-7}\ T)$
$= \boxed{45\ V/m}$

b) Ave. power per unit area = Intensity.

$$I = \frac{E_{MAX} B_{MAX}}{2\mu_0} = \frac{(45\ V/m)(1.5 \times 10^{-7}\ T)}{2(4\pi \times 10^{-7}\ Tm/A)}$$
$$= \boxed{2.7\ W/m^2}$$

13.

a) AM (540 – 1600 kHz): Since $\nu \propto \frac{1}{\lambda}$,
$\nu_{MIN} \rightarrow \lambda_{MAX}$ and $\nu_{MAX} \rightarrow \lambda_{MIN}$

($\nu \equiv$ Greek nu $\equiv$ frequency of E/M wave) $\left[Hz = \frac{1}{s}\right]$

$$\lambda_{MIN} = \frac{c}{\nu_{MAX}} = \frac{3.00 \times 10^8 \, m/s}{1600 \times 10^3 \, Hz} = \boxed{188 \, m}$$

$$\lambda_{MAX} = \frac{c}{\nu_{MIN}} = \frac{3.00 \times 10^8 \, m/s}{540 \times 10^3 \, Hz} = \boxed{556 \, m}$$

b) FM (88 – 108 MHz):

$$\lambda_{MIN} = \frac{c}{\nu_{MAX}} = \frac{3.00 \times 10^8 \, m/s}{108 \times 10^6 \, Hz} = \boxed{2.78 \, m}$$

$$\lambda_{MAX} = \frac{c}{\nu_{MIN}} = \frac{3.00 \times 10^8 \, m/s}{88 \times 10^6 \, Hz} = \boxed{3.4 \, m}$$

14.

a) From Coulomb's law: $F_e = \frac{k_e q_1 q_2}{r^2}$, $q_1 = q_2 = +e$

$$F_e = \frac{(8.99 \times 10^9 \, N \cdot m^2/C^2)(1.60 \times 10^{-19} \, C)^2}{(1.0 \times 10^{-15} \, m)^2} = \boxed{2.3 \times 10^2 \, N}$$

b) The electrical potential energy is: $PE = \frac{k_e q_1 q_2}{r}$

$$PE = \frac{(8.99 \times 10^9 \, Nm^2/C^2)(1.60 \times 10^{-19} \, C)^2}{1.0 \times 10^{-15} \, m} = 2.3 \times 10^{-13} \, J \left(\frac{1 \, MeV}{1.6 \times 10^{-13} \, J}\right) = \boxed{1.4 \, MeV}$$

15.

a) $r_n = n^2 a_0$, $r_2 = 4(0.0529\,nm) = \boxed{0.212\,nm}$

(Note that $1\,nm = 10^{-9}\,m$)

b) With electrical force supplying the centripetal acceleration, $F_c = F_e$, $q = e$, so

$$\frac{m_e v_n^2}{r_n} = \frac{k_e e^2}{r_n^2}, \quad v_n = \sqrt{\frac{k_e e^2}{m_e r_n}}$$

and $p_n = m_e v_n = \sqrt{\dfrac{m_e k_e e^2}{r_n}}$ = linear momentum

So, $p_2 = \sqrt{\dfrac{m_e k_e e^2}{r_2}}$

$$= \sqrt{\frac{(9.11\times10^{-31}\,kg)(8.99\times10^{9}\,Nm^2/C^2)(1.60\times10^{-19}\,C)^2}{0.212\times10^{-9}\,m}}$$

$$= \boxed{9.95\times10^{-25}\,kg\,m/s}$$

c) Angular momentum: $L_n = n\left(\frac{h}{2\pi}\right)$

$$L_2 = 2\left(\frac{6.63\times10^{-34}\,Js}{2\pi}\right) = \boxed{2.11\times10^{-34}\,Js}$$

d) $KE_n = \frac{1}{2} m_e v_n^2 = \dfrac{p_n^2}{2m_e}$

$$KE_2 = \frac{p_2^2}{2m_e} = \frac{(9.95\times10^{-25}\,kg\,m/s)^2}{2(9.11\times10^{-31}\,kg)}$$

$$= 5.43\times10^{-19}\,J\left(\frac{1\,eV}{1.60\times10^{-19}\,J}\right) = \boxed{3.40\,eV}$$

e) $PE_n = \frac{k_e q_1 q_2}{r_n}$, though here, we need to pay attention to the sign of the charges:

$$q_1 = -e \text{ (electron)}, \quad q_2 = e \text{ (proton)}$$

$$PE_2 = \frac{k_e(-e)e}{r_2} = -\frac{(8.99\times10^9\,Nm^2/C^2)(1.60\times10^{-19}\,C)^2}{0.212\times10^{-9}\,m}$$

$$= -1.09\times10^{-18}\,J\left(\frac{1\,eV}{1.60\times10^{-19}\,J}\right) = \boxed{-6.80\,eV}$$

f) Total energy: $E_n = KE_n + PE_n$

$$E_2 = KE_2 + PE_2 = 3.40\,eV - 6.80\,eV = \boxed{-3.40\,eV}$$

Since the energy is negative, the electron is <u>bound</u> to the proton as it orbits it.

16.

When the centripetal acceleration is supplied by the gravitational force, we have:

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \quad \text{or} \quad v^2 = \frac{GM}{r}$$

where here, $M \equiv$ mass of Sun, $m \equiv$ mass of Earth, $r \equiv$ separation of Sun-Earth.

3-15

a) With $PE = -\frac{GMm}{r}$, the total energy is

$$E = KE + PE = \frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{m}{2}\left(\frac{GM}{r}\right) - \frac{GMm}{r}$$

$$\boxed{E = -\frac{GMm}{2r}}$$

b) Using the Bohr quantization rule,

$$L_n = mv_n r_n = n\hbar, \text{ so } v_n = \frac{n\hbar}{mr_n}, \quad \hbar = \frac{h}{2\pi}$$

and $v^2 = \frac{GM}{r}$ becomes $\left(\frac{n\hbar}{mr_n}\right)^2 = \frac{GM}{r_n}$, or

$$r_n = \frac{n^2\hbar^2}{GMm^2} = \boxed{n^2 r_0}$$

with $r_0 = \frac{\hbar^2}{GMm^2} = \frac{(6.63\times10^{-34}\,Js)^2/4\pi^2}{(6.67\times10^{-11}\,Nm^2/kg^2)(1.99\times10^{30}\,kg)(5.98\times10^{24}\,kg)^2}$

$$r_0 = \boxed{2.32\times10^{-138}\,m}$$

c) The energy in the nth orbit is

$$E_n = -\frac{GMm}{2r_n} = -\frac{GMm}{2}\left(\frac{GMm^2}{n^2\hbar^2}\right) = \boxed{-\frac{E_0}{n^2}}$$

where

$$E_0 = \frac{G^2M^2m^3}{2\hbar^2} = \frac{(6.67\times10^{-11}\,Nm^2/kg^2)(1.99\times10^{30}\,kg)^2(5.98\times10^{24}\,kg)^3}{2(6.63\times10^{-34})^2/4\pi^2}$$

$$= \boxed{1.71\times10^{182}\,J}$$

d) $r_n = n^2 r_0$, so setting $r_n = 1$ A.U. we get

$$n^2 = \frac{r_n}{r_0} = \frac{1.49 \times 10^{11}\,m}{2.32 \times 10^{-138}\,m} = 6.42 \times 10^{148}$$

or $\boxed{n = 2.53 \times 10^{74}}$

e) No, since the quantum numbers are so large in this region, the allowed energies of orbits are essentially continuous.

17.

$$r_n = \frac{n^2}{Z}\left(\frac{\hbar^2}{m_e k_e e^2}\right) = \frac{n^2 a_0}{Z}, \text{ so } r_1 = \frac{a_0}{Z} = \frac{0.0529\,nm}{Z}$$

a) For $He^+ = He\,II$, $Z=2$, $r = \frac{0.0529\,nm}{2} = \boxed{0.0265\,nm}$

b) For $Li^{2+} = Li\,III$, $Z=3$, $r = \frac{0.0529\,nm}{3}$

$= \boxed{0.0176\,nm}$

c) For $Be^{3+} = Be\,IV$, $Z=4$, $r = \frac{0.0529\,nm}{4}$

$= 0.0132\,nm$