General Physics II Lab (PHYS-2021)
EXPERIMENT OPTC–2:
Mirrors and Image Formation

1 Introduction

A *spherical mirror* is a section of a spherical surface of radius R. There are two types of spherical mirrors as shown in Figure 1:

- **Concave mirror**: Reflecting surface is on the "inside" of the curved surface.
- **Convex mirror**: Reflecting surface is on the "outside" of the curved surface.

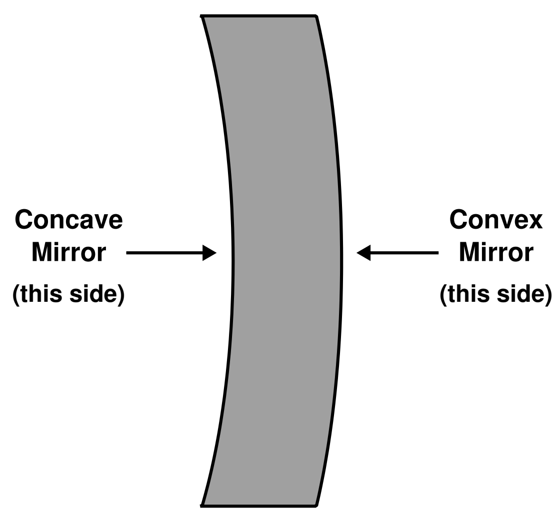


Figure 1: Two types of spherical mirrors.

Spherical mirrors may be used to form *images* from an *object* in the field of view of the mirror. In this laboratory experiment, you will learn how to construct an image for spherical mirrors using three different methods:

1. **Graphical ray-tracing method:** One makes a scale drawing on graph paper based on geometry.
2. **Analytic method:** One uses algebra and trigonometry, making use of the *mirror equation* and the *thin mirror equation*, to determine the location, orientation, and size of an image.
3. **Experimental method:** One uses mirrors, a source, and an image screen on an optics bench to determine the focal length (or radii of curvature) of the mirror. We will only deal with **concave mirrors** for the experimental method.

When dealing with image formation from mirrors, there are a variety of terms that we must define first.

- Image orientation:
  - **Erect Image**: Image is oriented the same as the object.
  - **Inverted Image**: Image is flipped 180° with respect to the object.
- Image classification:
  - **Real Image**: Image is on the same side of mirror as the object $\longrightarrow$ light rays actually pass through the image point.
  - **Virtual Image**: Image is on the opposite side of mirror from object $\Longrightarrow$ light rays appear to diverge from image point.
- Image size is determined by the magnification of an object which is given by

$$\boxed{M \equiv \frac{\text{image height}}{\text{object height}} = \frac{h'}{h}\ .} \tag{1}$$

$\lvert M\rvert > 1$	$\Longrightarrow$	Image is bigger than object (magnified).
$\lvert M\rvert = 1$	$\Longrightarrow$	Image is unmagnified (like a plane mirror).
$\lvert M\rvert < 1$	$\Longrightarrow$	Image is smaller than object (de-magnified).
$M > 0$	$\Longrightarrow$	Image is erect.
$M < 0$	$\Longrightarrow$	Image is inverted.
$M = 0$	$\Longrightarrow$	No image is formed.

In the box above, $|M|$ means the absolute value of the magnification M.

2 Image Formation with Concave Mirrors

Figure 2 shows a drawing of a concave mirror with various defining labels/markers of the mirror and an example of image formation for these mirrors.

- The line that is normal to the mirror surface at the exact center is called the **optical axis** of the mirror.
- The point where the optical axis intersects the mirror surface is called the **vertex** (labeled 'V' in the preceding diagram).

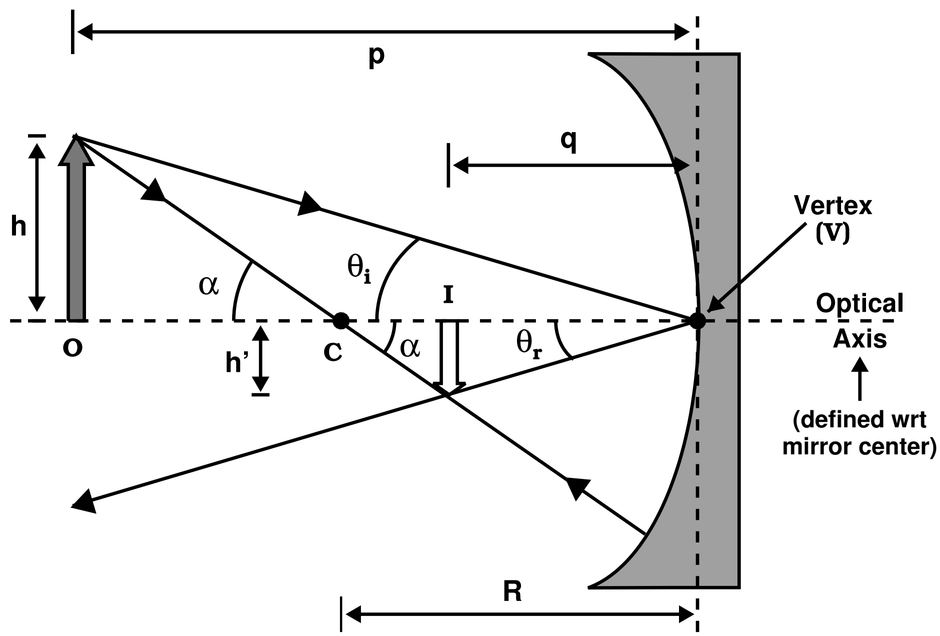


Figure 2: Image Formation Example for a Concave Mirror.

- Point 'C' indicates the position of the **center of curvature** of the mirror $\Longrightarrow$ line CV is equal to the **radius of curvature**, R, of the mirror.

- The **object** is labeled with 'O' and is designated with a 'filled' arrow in Figure 2. It is located at a distance p from the vertex on the optical axis of the mirror and has a height of h.
- The **image** is labeled with 'I' and is designated with a 'unfilled' arrow in Figure 2. It is located at a distance q from the vertex on the optical axis of the mirror and has a height of h'.
- Follow the principle ray from the tip of the object down to the vertex V. The angle that ray makes with the optical axis we will label as θ_i, where 'i' means 'incident' angle.
- That ray then gets reflected from the mirror at the vertex using the law of reflection. The angle between the reflected ray and the optical axis will be labeled as θ_r, where the 'r' means 'reflected' angle.
- To construct the image from that object, we use the law of reflection:

$$\theta_i = -\theta_r \ , \tag{2}$$

where θ is measured with respect to the normal of the mirror surface at the vertex V. The 'negative' sign is introduced here to note that the reflected angle sweeps away from the optical axis in the opposite 'sense' of the incident angle.

  – All normal lines on spherical concave mirrors go through center of curvature point C ($\theta_i = \theta_r = 0$)!

- Now look at the triangle involving sides h, p, and angle θ_i, and the triangle involving sides h', q, and angle θ_r in Figure 2. Using trigonometry, we see that

$$\frac{h'}{q} = \tan\theta_r \qquad \& \qquad \frac{h}{p} = \tan\theta_i \ .$$

  – Since $\theta_i = -\theta_r$, we get $\tan\theta_i = -\tan\theta_r$, and hence

$$\frac{h'}{q} = -\tan\theta_i = -\frac{h}{p} \quad \text{or}$$

$$\boxed{M = \frac{h'}{h} = -\frac{q}{p} \ .} \tag{3}$$

  – The magnification also can be determined by the ratio of the image to the object distance.

- Using the "α" triangles in the Figure 2, we can write

$$\tan\alpha = \frac{h}{p-R} \qquad \& \qquad \tan\alpha = -\frac{h'}{R-q} \ ,$$

$$\text{or} \quad \frac{h}{p-R} = -\frac{h'}{R-q} \ ,$$

$$\text{or} \quad \frac{h'}{h} = -\frac{R-q}{p-R} \ .$$

$$\text{Finally, using Eq. (3) gives} \quad \frac{q}{p} = \frac{R-q}{p-R} \ .$$

Solving this above expression gives

$$\begin{aligned} \frac{R-q}{q} &= \frac{p-R}{p} \\ \frac{R}{q} - 1 &= 1 - \frac{R}{p} \\ \frac{R}{q} + \frac{R}{p} &= 1 + 1 = 2 \ . \end{aligned}$$

Finally,

$$\boxed{\frac{1}{p} + \frac{1}{q} = \frac{2}{R}}\ , \tag{4}$$

which is the **mirror equation**.

- If $p \gg R$, then $1/p \ll 2/R$, so we say that as $p \to \infty$, $1/p \to 0$ and

$$\frac{1}{q} = \frac{2}{R} \quad \text{or} \quad q = \frac{R}{2}$$

  $\Longrightarrow$ the image is formed (*i.e.*, comes to a focus) halfway out to the center of curvature.

- So when $p \gg R$, the focal length of the mirror is

$$f = \frac{R}{2}\ . \tag{5}$$

- When $p \gg R$, the mirror appears "thin" to the distant object, therefore Eq. (5) is called the **thin mirror approximation** and we rewrite Eq. (4) as

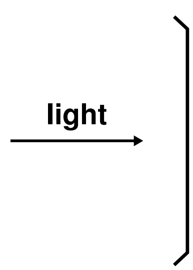
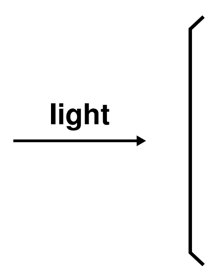
$$\boxed{\frac{1}{p} + \frac{1}{q} = \frac{1}{f}}\ . \tag{6}$$

- Both convex and concave mirrors use Eq. (6), except there is a "change" in sign for the radius and focal length of the mirror. Table 1 shows the sign conventions used for the geometric optics parameters for curved mirrors.

- Image location can either be determined algebraically from Eqs. (3) & (6), by drawing ray diagrams, or by experimental methods. There are **three principle rays** that define the image location for **concave mirrors** as shown in Figures 3:

  - Ray 1 is drawn parallel to the optical axis and is reflected back through the focal point, F.
  - Ray 2 is drawn through the focal point, F, and reflected parallel to the optical axis.
  - Ray 3 is drawn through the center of curvature, C, and reflected back on itself.

Table 1: **Sign Conventions for Curved Mirrors**

	SIGNS	
	PLUS (+)	**MINUS (–)**
p	object left of mirror (real object)	object right of mirror (virtual object)
q	image same side of mirror as object (real image)	image opposite side of mirror as object (virtual image)
h	object is erect	object is inverted
h'	image is erect	image is inverted
M	image is in same orientation as object	image is inverted with respect to object
R	concave mirror	convex mirror
f	concave mirror	convex mirror
symbol		

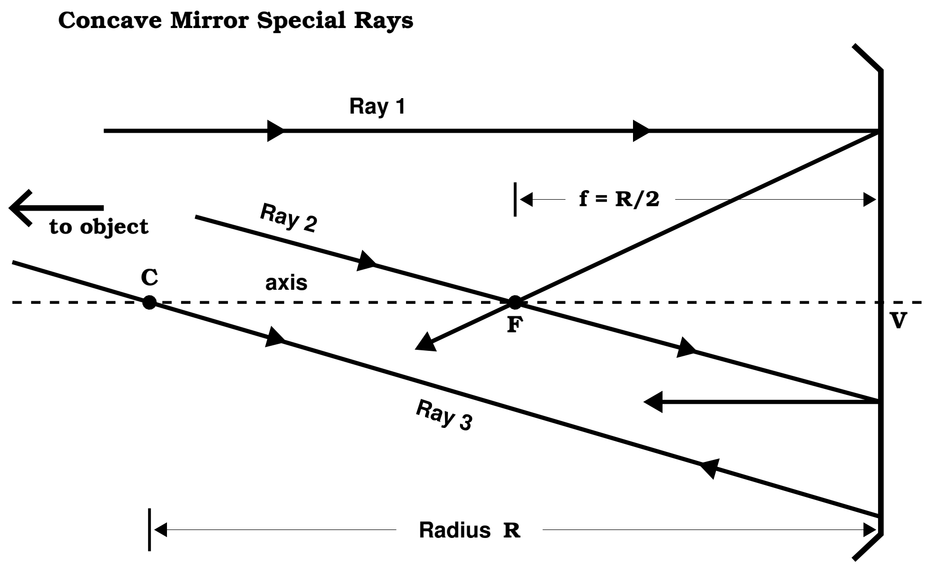


Figure 3: Three Principle Rays of a Concave Mirror.

3 Image Formation with Convex Mirrors

The equations that we developed for concave mirrors are also valid for convex mirrors except the signs for the various parameters are opposite of what they were for concave mirrors as shown in Table 1. Just as we had for concave mirrors, we can define **three principle rays** for **convex mirrors** as shown in Figure 4:

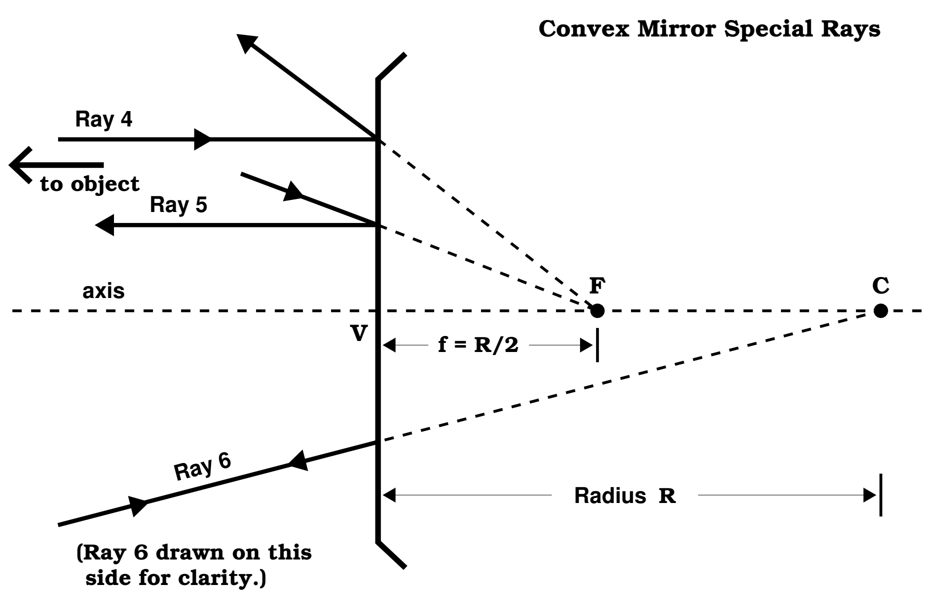


Figure 4: Three Principle Rays of a Convex Mirror.

- Ray 4 is drawn parallel to the optical axis and is reflected back away from the focal point, F, on the back side of the mirror.
- Ray 5 is drawn toward the focal point, F, on the back side of the mirror and reflected back, parallel to the optical axis.
- Ray 6 is drawn toward the center of curvature on the back side of the mirror, C, and reflected back on itself.

4 Analytic Examples for Image Formation of Spherical Mirrors

1. A 2.50 cm tall object is placed 7.20 cm in front of a concave mirror of radius 8.00 cm. Locate and describe the image.

   First we associate the given parameters with their variables:

Object Height:	$h = +2.50$ cm
Object Distance:	$p = +7.20$ cm
Mirror Radius:	$R = +8.00$ cm (concave mirror)

   Using the mirror equation, we now solve for the distance that the image is from the mirror:

$$\begin{aligned} \frac{2}{R} &= \frac{1}{p} + \frac{1}{q} \\ \frac{1}{q} &= \frac{2}{R} - \frac{1}{p} \\ &= \frac{2}{8.00\text{ cm}} - \frac{1}{7.20\text{ cm}} \\ &= 0.250\text{ cm}^{-1} - 0.139\text{ cm}^{-1} = 0.111\text{ cm}^{-1} \\ q &= +9.00\text{ cm.} \end{aligned}$$

   Now we calculate the magnification and from that, calculate the image height:

$$\begin{aligned} M &= -\frac{q}{p} = -\frac{9.00\text{ cm}}{7.20\text{ cm}} = -1.25 \\ M &= \frac{h'}{h} \\ h' &= h * M = 2.50\text{ cm} * (-1.25) \\ &= -3.1\underline{2}5\text{ cm} = -3.13\text{ cm.} \end{aligned}$$

$\therefore$ Hence we have a <u>real</u> ($q > 0$), <u>inverted</u> ($h' < 0$), and <u>magnified</u> ($|M| > 1$) image.

2. A 2.50 cm tall object is placed 1.80 cm in front of a <u>concave</u> mirror of radius 8.00 cm. Locate and describe the image.

   Just as we did for #1, we associate the given parameters with their variables:

Object Height:	$h = +2.50$ cm
Object Distance:	$p = +1.80$ cm
Mirror Radius:	$R = +8.00$ cm (concave mirror)

Using the mirror equation, we now solve for the distance that the image is from the mirror:

$$\begin{aligned} \frac{2}{R} &= \frac{1}{p} + \frac{1}{q} \\ \frac{1}{q} &= \frac{2}{R} - \frac{1}{p} \\ &= \frac{2}{8.00 \text{ cm}} - \frac{1}{1.80 \text{ cm}} \\ &= 0.250 \text{ cm}^{-1} - 0.55\underline{5}6 \text{ cm}^{-1} = 0.30\underline{5}6 \text{ cm}^{-1} \\ q &= -3.27 \text{ cm}. \end{aligned}$$

Now we calculate the magnification and from that, calculate the image height:

$$\begin{aligned} M &= -\frac{q}{p} = -\frac{(-3.27 \text{ cm})}{1.80 \text{ cm}} = +1.82 \\ M &= \frac{h'}{h} \\ h' &= h * M = 2.50 \text{ cm } * (1.82) \\ &= +4.5\underline{4}5 \text{ cm } = +4.55 \text{ cm}. \end{aligned}$$

$\therefore$ Hence we have a <u>virtual</u> ($q < 0$), <u>erect</u> ($h' > 0$), and <u>magnified</u> ($|M| > 1$) image.

3. A 4.20 cm tall object is placed 4.80 cm in front of a <u>convex</u> mirror of radius 8.50 cm. Locate and describe the image.

   Once again, we associate the given parameters with their variables:

Object Height:	$h = +4.20$ cm
Object Distance:	$p = +4.80$ cm
Mirror Radius:	$R = -8.50$ cm (convex mirror)

Using the mirror equation, we now solve for the distance that the image is from the mirror:

$$\begin{aligned} \frac{2}{R} &= \frac{1}{p} + \frac{1}{q} \\ \frac{1}{q} &= \frac{2}{R} - \frac{1}{p} \\ &= \frac{2}{-8.50 \text{ cm}} - \frac{1}{4.80 \text{ cm}} \\ &= -0.23\underline{5}3 \text{ cm}^{-1} - 0.20\underline{8}3 \text{ cm}^{-1} = -0.44\underline{3}6 \text{ cm}^{-1} \\ q &= -2.2\underline{5}4 \text{ cm} = -2.25 \text{ cm}. \end{aligned}$$

Now we calculate the magnification and from that, calculate the image height:

$$\begin{aligned} M &= -\frac{q}{p} = -\frac{(-2.2\underline{5}4 \text{ cm})}{4.80 \text{ cm}} = +0.470 \\ M &= \frac{h'}{h} \\ h' &= h * M = 4.20 \text{ cm} * (0.470) \\ &= +1.9\underline{7}2 \text{ cm} = +1.97 \text{ cm}. \end{aligned}$$

$\therefore$ Hence we have a virtual ($q < 0$), erect ($h' > 0$), and demagnified ($|M| < 1$) image.

5 Procedure

5.1 Experimental Procedure

Table 2: Items Used in the Mirrors and Image Formation Lab.

1.2 meter optical bench (OS-8508)	light source (OS-8517)
100 mm concave mirror (OS-8457)	half-screen (OS-8457)
metric ruler/straight edge	graph paper

PASCO Part Number in parentheses.

Note that in this lab, we will only be working with **concave mirrors** in the experimental portion of this lab. The purpose of this experiment is to measure the focal length of a concave mirror.

Caution! *Be careful not to touch or scratch the surface of the mirror!*

Caution! *Be sure not to block the lamp-housing circular air vent at the top of the housing! If blocked, the housing will overheat and burn out the light bulb and transformer.*

Note: It is important that you properly align the light source, mirror and the image screen each time you do an experimental setup, *i.e.*, the mirror is approximately perpendicular to the bench axis, and the screen is positioned properly to capture the image. Some trial-and-error efforts are usually required to capture the best alignment.

The Concave/convex Mirror Accessory is designed for use with an PASCO optics bench (OS-8508). It consists of a double- sided curved mirror (OS-8457) and a semicircular half-screen (OS-8457). The mirror is concave on one side and convex on the other side. We will only be working with the concave side in the experimental portion of this lab. Both sides have the same radius of curvature. See Figure 5 for a drawing of the experimental set-up for this lab.

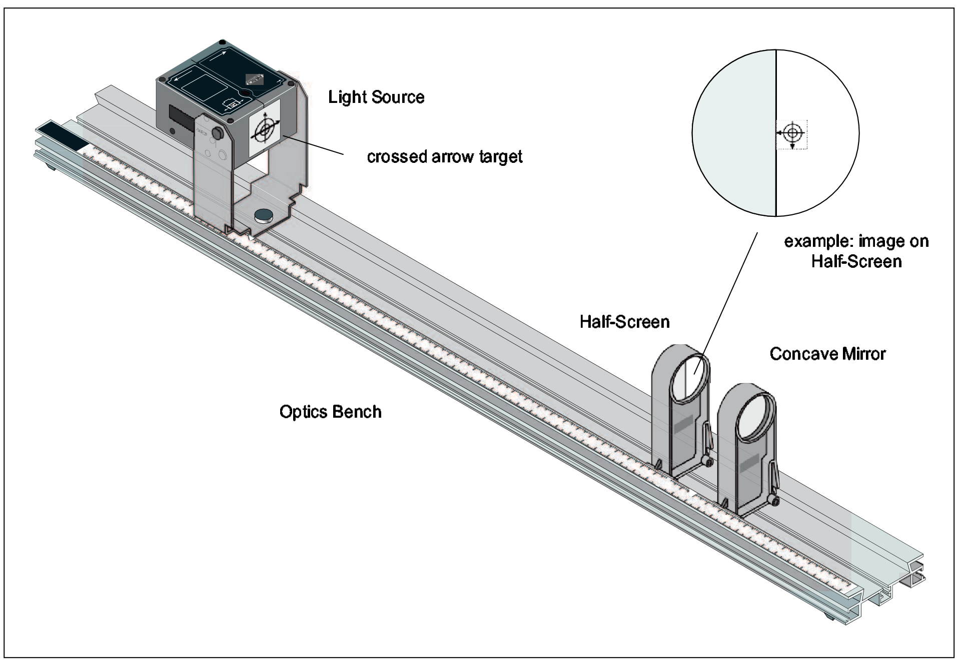


Figure 5: Experimental set-up for the concave spherical mirror experiment.

The half-screen is designed for showing an image formed by the concave side of the mirror. The screen on the half-screen can be rotated in its mount so that its edge is vertical or horizontal as shown in Figure 6.

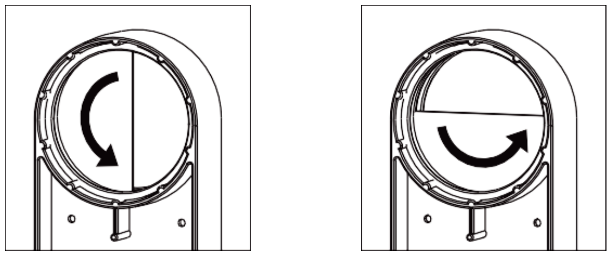


Figure 6: Two possible orientations for the half-screen.

The concave mirror accessory and the half-screen accessory snap into place on the Optics Bench. To move an accessory along the bench, grasp the base of the accessory holder and squeeze the locking clip inward. Continue to squeeze inward on the locking clip as you move the accessory to the new position. When you release the locking clip, the accessory holder is held firmly in place.

Experimental Steps:

You will be making 3 separate measurements for these steps and write the measurements in a data table. Once the 3 measurements are made, you will calculate the average of the measurements and determine the uncertainty in your averages.

1. Mount the Light Source at one end of the Optics Bench. Place the Concave Mirror at the other end of the Optics Bench as shown in Figure 5.
2. Position the Light Source so the crossed arrow target on the light source is aimed at the Concave Mirror and the concave surface of the mirror faces the light source.
3. Place the Half-Screen a few centimeters in front of the Concave Mirror (between the mirror and the Light Source).
4. Move the Half-Screen closer to or farther from the Concave Mirror until the reflected image of the crossed arrow target on the white screen is focused.
5. Measure the distance between the crossed-arrow object slide on the light source and the position indicator on the Concave Mirror – this will be your object distance.
6. Measure the distance between the position indicators on the Half-Screen and the Concave Mirror – this will be your image distance.

7. Measure the size of the object on the crossed-arrow object slide on the Light Source box and the size of the image on the half-screen using your metric ruler. Do the best you can when measuring the image size on the half-screen.

Once you are through with your measurements, calculate the focal length of mirror using Eq. (6). Then use Eq. (4) to calculate the radius of curvature. Following this, calculate the magnification of the image using the ratio of h' over h in Eq. (3), then by the negative ratio of q and p in this equation. Include these calculations in your Lab Report.

Experimental Questions:

1. How does the measured distance compare to the focal length of the Concave Mirror?
2. How might you determine the focal length more accurately?
3. What is the orientation of the image of the crossed arrow target compared to the target itself?
4. How does the size of the image of the crossed arrow target compare to the target itself? Compare your h' and h ratios with your negative q and p ratios. Which of these two ratios is more likely to be the more accurate magnification value and why do you say this?

5.2 Analytic and Graphical Procedure

(**Note that this step can be completed at home.**) For each of the following problems, you are to (1) **construct ray diagram** on graph paper to *locate* and *describe* the image, and (2) **analytically** determine the image *location* (supplying proof whether the image is real or virtual), *image size* and *magnification* (supplying proof whether the image is upright or inverted, and magnified or de-magnified). Work carefully, accurately, and neatly; use a sharp pencil, an accurate ruler, and a good straight edge (typically on the ruler) to draw your light rays. Make sure you pay attention to significant digits for the given input parameters and the results you calculate and determine from your graphs. Be sure to *fully label each diagram:* p, q, f, O (the object), I (the image), h, h', etc. Indicate a scale factor on your graphs, for instance, each bold-line square corresponds to 2.00 cm. Finally by convention, ray diagrams are drawn with the incident light traveling from left-to-right so that p (the objects distance from the mirror) is a positive number.

1. An object is 4.30 cm high and is placed 12.6 cm to the left of a concave mirror. The mirror has a radius of curvature of 6.20 cm. Locate and describe the image.

2. An object is 3.60 cm high and is placed 8.20 cm to the left of a concave mirror. The mirror has a radius of curvature of 10.2 cm. Locate and describe the image.

3. An object is 5.50 cm high and is placed 3.30 cm to the left of a convex mirror. The mirror has a radius of curvature of 5.20 cm. Locate and describe the image.

4. An object is 3.62 cm high and placed 12.2 cm in front of a mirror. A real and inverted image of height (–)2.00 cm is formed. (a) Would a concave or convex mirror be required to form this image? Mathematically prove this answer. (b) What is the radius of curvature of this mirror?

5. Using algebra with the thin-mirror approximation (*i.e.*, Eq. 6), prove that the image produced by a convex mirror is always formed behind the mirror as a virtual image. (Please note that an *example* is not a sufficient proof!)

6. A 2.00 cm tall object is placed 4.00 cm in front of a concave mirror of radius 8.00 cm. Locate and describe the image.

6 Final Note

This Laboratory Experiment will require a standard **Lab Report** as described in the lab_report_format.pdf document on the course web page at
https://faculty.etsu.edu/lutter/courses/phys2021/index.htm.