PHYS-4007/5007 COMPUTATIONAL PHYSICS PROJECT: Projectile Motion with Air Resistance

1 Introduction

Following the material found in Section 15.10 of your textbook (page 228 of the 2nd Revised and Enlarged Edition) on *Projectile Motion with Drag* and Section IX.B.1 in the course notes, write either a Fortran or an IDL code to solve Newton's 2nd law of motion (a second order ODE), see Eq. (IX-30) in the notes, by breaking up this second-order equation into a set of first-order ODEs (see Eq. IX-43) using the 4th-order Runge-Kutta technique (either use the Fortran RK4 routine I will supply on the course web page (e.q., rk4_projmotion.f) or use the IDL RK4 function).

Your code should have the ability to accept the body's initial velocity, projection angle, time step, drag velocity exponent, and drag coefficient (see below).

Assume your projectile is spherical in shape with a surface area of $1.00~\rm{m^2}$ and has a mass of $10.0~\rm{kg}$. Note that these parameters actually are not needed in the RK4 calculation. Carry out calculations using the following information:

- Make sure all input and internal numbers are double precision.
- Assume the projectile is launched from the ground and lands on the ground (i.e., y = 0 m).
- For the drag force exponent, just carry out runs with n=2.
- For the size of our spherical object, use 0.470 m^{-1} for the drag coefficient k. Assume the drag coefficient is constant with height.
- Use the following initial velocities from ground level: 50.0 m/s and 200 m/s (assume 3 significant digits).
- For each velocity above, use the following projection angles: 32.0° and 56.0°.
- Try a variety of different time steps to investigate how the results change.

2 Analysis

Your analysis should include plots of y (vertical height) as a function of x (horizontal distance), y as a function of t (time), v_x as a function of t, and v_y as a function of t. You should also determine the maximum time step that produces a converged solution and explain how you determined this.

Helpful Items of Note: The IDL function RK4 has the following form:

```
Result = RK4(Y, DYDT, T, TAU, Derivs)
```

Here Y will be a 4-elements array containing $[x, v_x, y, v_y]$, DYDT will be a 4-element array containing the time-derivatives of these parameters $[v_x, a_x, v_y, a_y]$, T is the current time, TAU $(=\Delta t)$ is the current time step, and Derivs is the IDL function that you write that contains the equations for DYDT (see Eq. XII-39 through Eq. XII-42 in the notes). The output is stored in Result which will contain the new values of $[x, v_x, y, v_y]$ for the next time T+TAU, where the units of T and TAU are units of time. All numbers and variable should be written in double precision to reduce round-off error. The IDL RK4 function should be placed in a FOR loop and as the loop steps through time and x, y, v_x, v_y , and t should be stored in an array once they are calculated for each time step. The stored data can then be used to make your plots. For instance,

```
... precalculation coding ...
NTOT = 10000
                \% Enough points to be on the safe side.
XDIST = DBLARR(NTOT)
YHEIGHT = DBLARR(NTOT)
VX = DBLARR(NTOT)
VY = DBLARR(NTOT)
TIME = DBLARR(NTOT)
TIME[0] = 0.D0
XDIST[0] = 0.D0
YHEIGHT[0] = 0.D0
VX[0] = V0 * COS(THETA0)
                            % THETA0 in radians.
VY[0] = V0 * SIN(THETA0)
                            % THETA0 in radians.
ISTEP = 1
WHILE ISTEP GT 0 DO BEGIN
TIME[ISTEP] = TIME[ISTEP-1] + TAU
   ... DYDT function call ...
   ... RESULT = RK4() function call ...
   ILAST = ISTEP
          (continued on next page)
```

```
IF RESULT[1] GE 0. THEN BEGIN

X[ISTEP] = RESULT[0]

VX[ISTEP] = RESULT[1]

Y[ISTEP] = RESULT[2]

VY[ISTEP] = RESULT[3]

ISTEP = ISTEP + 1

ENDIF ELSE ISTEP = -1

IF ISTEP GE NTOT THEN ISTEP = -1

ENDWHILE

... now do the plotting ...
```

3 Writing the Final Manuscript

The Final Manuscript must be at least 5 pages in length (not including references, tables, figures, and code listing) for undergraduate students and 10 pages in length for honors and graduate students. Your references, figures, and tables should take up no more than an additional 5 pages. Note that your manuscript must be written in LATEX! You will need to follow the same style used in profession scientific journals.

Feel free to use the professional journal style LaTeX template file (i.e., template.tex) for your manuscript. Email the LaTeX file of your manuscript, the PDF file generated from it, and your code(s) to me by the project due date.