PHYS-4007/5007
COMPUTATIONAL PHYSICS PROJECT:
Data Fitting to Hubble's Law

1 Introduction

This Computer Class Project will involve writing an IDL computer code dealing with data fitting and analysis. A web site has been made that contains spectra of a set of galaxies (in ASCII format with files ending in ".txt"), an ASCII file containing the distances and distance uncertainties of the galaxies in the sample (called "galdist.txt"), a *nearly completed* IDL procedure (called "hubble.pro") that you can use to carry out this project, and a copy of this document written in LaTeX, which you can use as a template for writing up your manuscript for this project. **Note that you should not store the galaxy distance file (galdist.txt) in the same directory as the spectrum *.txt files.**

For this project, you will be determining **Hubble's Constant**, $H_\circ$, which is used in Hubble's Law:

$$v_r = H_\circ \, d \ ,$$

where v_r is the radial velocity (in km/s) of the galaxy in question and d is the distance to the galaxy (in Mpc = 10^6 parsecs). Note that the "parsec" is a unit of distance used in astronomy and is equal to 3.26 light years — an astronomical object that displays a yearly parallax shift of one arcsecond would be at a distance of one parsec as per the equation

$$d = \frac{1}{p} \ ,$$

where d is the distance measured in parsecs and p is the parallax angle measured in arcseconds (though you will not need this parallax equation for this project).

This Instruction Packet contains a short description of Hubble's Law, information on what you need to figure out in your program, the techniques and IDL functions and procedures are used in the procedures, and an outline on how you should format your manuscript. Those of you taking this course for honors or graduate credit will be expected to carry out additional work as will be described in §6 of this Instruction Packet.

The "hubble.pro" code will generate both a normal ("hubblelaw.ps") and an encapsulated ("hubblelaw.eps") **postscript** hardcopy plot showing your reduced data and the fitted line in addition to the plot displayed on the terminal. Note that the encapsulated plot can be imported into the $\LaTeX$ file you will need to write for this project (see §6 of this Instruction Packet).

2 Hubble's Law and Hubble's Constant

Astronomers determine the distances to galaxies by using various **distance indicators**:

- Galaxies that are at a distance of $d < 6$ Mpc:
  - Classical Cepheids and W Virginis pulsating stars ($P \rightarrow m - M_V \rightarrow d$). The star's pulsation period is related to the luminosity of the star, where here, P, is the period of the variable star's light curve, m is the apparent magnitude (brightness seen at Earth) of the pulsating star, and M_V is the absolute magnitude (brightness as seen from a distance of 10 pc) of the star. Then the distance d to these stars can be determined from these magnitudes with the distance modulus formula:

    $$m - M_V = 5 \log d - 5 \ ,$$

    where d is measured in parsecs in this equation. Note that these types of variable stars are very luminous and hence can be seen from very large distances.
  - O & B main sequence stars. The stars are very luminous and have known luminosities.
  - Novae. These objects result from stellar material being dumped onto a white dwarf (degenerate matter) star which results in a runaway thermonuclear explosion on the WD's surface. When they explode, they all reach the same brightness.
  - RR Lyrae stars. Another type of variable, where all have the same approximate luminosity.
  - For each of these objects, their luminosity is fairly well known, and they are bright, meaning they can be seen to fairly great distances. Their luminosities are compared to the brightness as seen from Earth, and using the inverse-square law of light, their distances determined.
  - Hence, looking for these stars in external galaxies will indicate the distance to that galaxy.
- Galaxies that are at distances between $4 < d < 40$ Mpc:
  - Diameter of giant H II (ionized hydrogen) regions (all have the same approximate size).
  - Brightness of largest globular star clusters (all have the same approximate brightness).

- Galaxies that are at distances between $40 < d < 600$ Mpc:
  - Tully-Fisher relation: The FWHM (Full-Width-at-Half-Maximum) of the Gaussian profile of the H I 21-cm line is proportional to the luminosity of an elliptical galaxy.
  - Brightest of Sc I (spiral galaxies with loose arms) galaxies.
  - Supernovae, which are supermassive stars that blow apart as their core finishes its thermonuclear lifetime and collapses to form a neutron star or black hole.
  - The 3 brightest galaxies in a cluster.
  - Diameters of bright galaxies.
- Galaxies that are at a distance $d > 600$ Mpc: Use Hubble's Law.
- Each of these distance indicators is calibrated on the previous ones!

In the 1920's, Edmund Hubble compared magnitudes of galaxies to their redshifts and found that the fainter the galaxy, the bigger the redshift $\Longrightarrow$ **Hubble's Law**. The more distant a galaxy (*i.e.*, fainter galaxies), the larger the redshift hence recession velocity

$$\Longrightarrow \boxed{\textbf{The Universe is Expanding.}}$$

Hubble's Law mathematically is simply a linear relation between the distance, d to a galaxy and its radial (line-of-sight) velocity, v_r:

$$v_r = H_\circ \, d \; .$$

The slope of the line representing this linear relation has become known as Hubble's Constant and is represented by the symbol $H_\circ$ (the "$\circ$" means current value). Up until just the last decade, $H_\circ$ was not accurately known (various studies put in the range 50–100 km/sec/Mpc) $\Longrightarrow$ its actual value is of the utmost importance! The primary reason the *Hubble Space Telescope* was built was to determine an accurate value for $H_\circ$. **It will be your job to determine $H_\circ$ from actual galactic spectra!**

Another parameter that is often used in cosmology is the redshift, z, which is defined as

$$z \equiv \frac{\Delta\lambda}{\lambda_\circ} = \frac{v_r}{c} \; ,$$

where we also include the *nonrelativistic* ($v_r \ll c$) form of the **Doppler Effect** ($\Delta\lambda/\lambda_\circ = v_r/c$). We also can write a more general expression for the redshift,

$$z = \frac{\Delta\lambda}{\lambda_\circ} = \frac{\sqrt{1 + v_r/c}}{\sqrt{1 - v_r/c}} - 1 \; .$$

where, here, we have included the *relativistic* (*i.e.*, $v_r \lesssim c$) correction for the Doppler Effect. Rewriting this relativistic formula, we can express velocity as a function of redshift:

$$\frac{v_r}{c} = \frac{(z+1)^2 - 1}{(z+1)^2 + 1} \ ,$$

You will note that there is no way for a galaxy's velocity to exceed that of light when using the relativistic form of the Doppler Effect.

Prior to Hubble's discovery, the Universe was assumed to be static, infinite, and eternal. From this assumption, it was reasoned that we should light in every direction we look $\Longrightarrow$ **Olber's Paradox**. But Hubble showed that the Universe is expanding (hence not static) and is not eternal, hence the assumptions in Olber's paradox are invalid. Since the galaxies are moving apart from each other, there must have been some time in the past when the galaxies were fairly close together. This thought gave rise to the **Big Bang Theory** — it had a beginning! As a result of this, light gets redshifted out of the visible band. Also, as we look out, we look back in time. We cannot look infinitely far out since, sooner or later, we will see the Big Bang (and indeed we do see the Big Bang as the *cosmic microwave background radiation*)!

Let's ask the question, how long ago did this happen, that is, when were all of the galaxies close together? Initially, we will treat this question in a very simplified way. Using simple mechanics, an object will travel a distance d going at a certain constant velocity v in a time period T given by $d = v\,T$. Here, we are using d as the distance to a galaxy, v as the velocity that the galaxy is traveling, and T is the time since the galaxy started from the origin. But from Hubble's Law, $v = H_\circ\, d$, so

$$T = \frac{d}{v} = \frac{d}{H_\circ\, d} = \frac{1}{H_\circ}$$

From this simple exercise, we need to point out that the Universe is actually younger than that due to gravity slowing down the expansion over time — this time corresponds to a maximum age of the Universe. This **Hubble Time** can be expressed in terms of the measured Hubble constant via

$$\boxed{T_\circ[\text{years}] = \frac{978 \times 10^9}{H_\circ[\text{km/s/Mpc}]} \ .}$$

If $T = 15$ billion years, then a galaxy farther than 15 billion light years away, we will never seen at the present time since light would not have had enough time to reach us $\Longrightarrow$ 15 billion light years would be the size of the **observable universe**.

Continuing on with our Newtonian description of the Universe, for Hubble's Law (the observed *kinematic world model*) to be true, galaxies in the Universe have to be distributed *homogeneously* (matter is uniformly distributed in space) and *isotropically* (the Universe looks the same in every direction) on a large scale. E.A. Milne and W.H McCrea (1934) extended this, at first purely kinematic model, so as to make it a **Newtonian cosmology**. They investigated the motions of a medium (the *gas* of galaxies, *i.e.*, treat galaxies as point particles) that can take place in accordance with Newtonian mechanics if one demands throughout *homogeneity, isotropy*, and *irrotational* motion.

Consider at time t, a galaxy at distance $R(t)$, then according to Newton's law of gravitation, this galaxy is attracted by the mass within a sphere of radius R given by mass

$$M = (4\pi/3)\, R^3\, \rho(t),$$

where $\rho(t)$ is the mass-density at the instant considered. Thus, the equation of motion of this galaxy is determined by setting the force of motion equal to the gravitational force:

$$\frac{d^2R}{dt^2} + \frac{GM}{R^2} = 0,$$

where the mass M is constant. Multiplying each term in this equation by $\dot{R} = dR/dt$, it is then possible to easily integrate this equation and obtain the *energy equation*:

$$\frac{1}{2}\left(\frac{dR}{dt}\right)^2 - \frac{GM}{R} = h,$$

where h is the integration constant, or

$$\frac{\dot{R}^2}{R^2} - \frac{8\pi}{3}G\rho(t) + \frac{kc^2}{R^2} = 0,$$

in which we have written $-h = kc^2/2$ in anticipation of a comparison with relativistic calculations. From this equation, it can be seen that the Universe cannot remain static if $\rho > 0$ at any time.

We can define the current **Hubble Constant** as

$$H_\circ = \dot{R}_\circ / R_\circ,$$

where we denote present time $t = t_\circ$ by a subscript $\circ$. In reality, Hubble's Constant is not constant at all, and should be expressed as the **Hubble parameter**, $H(t)$, where $H_\circ$ is the current value of $H(t)$, or

$$H(t) = \frac{\dot{R}(t)}{R(t)} \ .$$

Using this in the equation of motion we wrote above, we can write a new equation of motion based on the Hubble parameter:

$$\frac{dH}{dt} + H^2 = \frac{-4\pi}{3}G\rho(t) \ .$$

Likewise, from the energy equation, we can write

$$H(t)^2 = \frac{8\pi}{3}G\rho(t) - \frac{kc^2}{R^2} \ .$$

As can be seen, the value of Hubble's "constant" changes over time and this change is a function of the density of the Universe at that given time.

For a complete characterization of a model universe a the current epoch, we need, besides $H_\circ$, a second variable that describes the deceleration of the Universe due to its mass M. This is the so-called **deceleration parameter**:

$$q_\circ = -\left(\frac{\ddot{R}_\circ}{R_\circ}\right) / \left(\frac{\dot{R}_\circ}{R_\circ}\right)^2 = -\frac{\ddot{R}_\circ}{R_\circ\, H_\circ^2} = \frac{4\pi G \rho_\circ}{3H_\circ^2} \ .$$

This formula relates the acceleration $\ddot{R}_\circ$ to a uniform acceleration which would lead to the observed velocity $R_\circ H_\circ$ at distance $R_\circ$ in the Hubble time $T_\circ = H_\circ^{-1}$, starting from zero velocity. The solution of the above equations leads to world models which, from a starting point (singularity) of infinitely great density, either expands monotonically (total energy $h \geq 0$) or oscillates periodically between $R = 0$ and an $R_{\max}$ (if $h < 0$). Static models are not possible within the framework of the $\dot{R}^2/R^2$ equation above.

Things get quite a bit more complicated when one takes general relativity into account. See my Astrophysics (ASTR-3415) course notes at

http://faculty.etsu.edu/lutter/courses/astr3415/

for details.

3 The *Hubble's Law* Project Web Site

I have linked the Course Project web page on the course home page — the full address of this Course Project web site is

http://faculty.etsu.edu/lutter/courses/phys4007/project/hubble.htm

All of the files that are described in this Instruction Packet can be found on this web page, including all of the galaxy spectra you need to download which were originally published by McQuade, Calzetti, & Kinney (1995) — note that you can download this paper too from this web site.

3.1 IDL Procedures to Download from the Project Web Site

You will need to download all of the IDL procedures tabulated on the Course Project web site listed in the following table. Note that you should store these files on your local machine in a subdirectory/folder specifically made for this Hubble project – let's assume this subdirectory is called "hubble."

hubble.pro:	The main driving procedure that does most of the work.
decideopt.pro:	GUI widget procedure used when a Gaussian fit is bad.
galflscale.pro:	Determine the flux scale factor for spectrum plots.
rdgaldist.pro:	Reads the galdist.txt galaxy distances file.
rdgalspec.pro:	Reads the galaxy spectrum files.
selectgal.pro:	Allows user to preselect spectra to be reduced.
spectype.pro:	GUI widget to select the type of galaxy spectrum.

Note that the **selectgal.pro** procedure is still under development and will be uploaded to the web site within the next week.

3.2 Data Files to Download from the Course Project Web Site

There are 35 data files, all with a filename suffix of ".txt," that you will need to download from the Course Project Web Page. One of the files, called galdist.txt, contains the distance data and the distance uncertainty data for the galaxies. This file should be located in the same directory as you IDL procedures. The remaining 34 ".txt" data files contain the galaxy spectra, one galaxy spectrum per file. These should be placed in a separate subdirectory/folder from your IDL procedures, say "spectra" in the "hubble" subdirectory. **Again, make sure that the "galdist.txt" file is not located in the same directory as the spectrum data files.**

4 Description of the Project

Once these procedures have been downloaded, students will have two write two lines of code in **hubble.pro** to calculate the normalized uncertainties for the distances data and the slope of the fitted straight line for Hubble's Law — see the "!!!!! STUDENT'S WRITTEN SECTION:" in the **hubble.pro** procedure. This will need to be done before you can run the code. See §4.7 for more details on how to do this.

Table 1: **Emission and Absorption Lines to Measure**

Emission Lines		**Absorption Lines**	
Line ID	**λ (Å)**	**Line ID**	**λ (Å)**
H-β	4861.3	Ca II K	3933.7
[O III]	4958.9	Ca II H	3968.5
[O III]	5006.9		

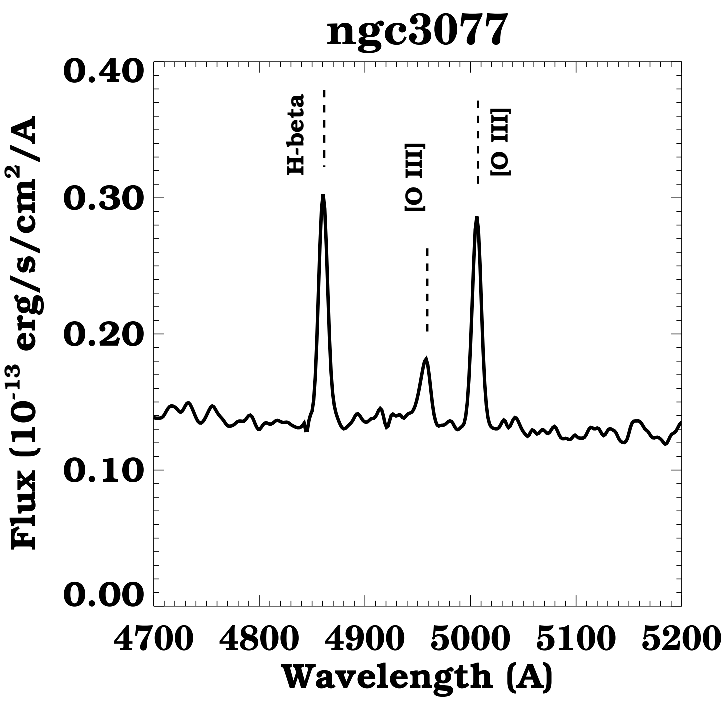

Figure 1: **Emission lines in galaxy NGC 3077.**

4.1 Basics of the hubble.pro Procedure

The bulk of the coding in the hubble.pro procedure involves determining the radial velocity of galaxies from their spectra. There are two types of spectra that you will see, either a pure **absorption line** spectrum or an **emission line** spectrum. Note that the emission line spectra also have absorption lines in them, but emission lines are also present and are very obvious in appearance. Line identifications and wavelengths for the emission and absorption lines you will be measuring are already included in the code. These lines are posted in Table 1 and shown in the spectrum of the emission-line galaxy NGC 3077 (Figure 1) and absorption-line galaxy NGC 244 (Figure 2). I made these *encapsulated postscript* figures with the IDL procedure mkproj1figs.pro which I wrote. I already have included the coding to make figures in the hubble.pro procedure.

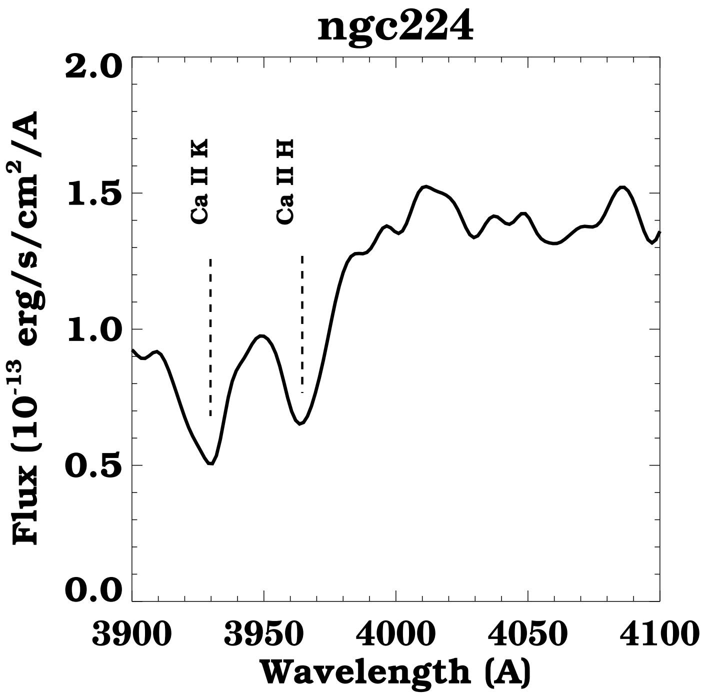

Figure 2: **Absorption lines in galaxy NGC 244.**

4.2 Read the Distance Data from an External Data File

I have given you a data file called galdist.dat that contains the names, distances (in Mpc), and the uncertainties (in Mpc) of the galaxies for this study. I have supplied you with a procedure (rdgaldist.pro) that will read this data file. The call to this procedure is already included in the hubble.pro procedure.

4.3 Reading the Galaxy Spectrum Files

The spectra on the Course Computer Project web page were downloaded from the web site

http://www.stsci.edu/ftp/catalogs/galaxy-spectra/

and these spectra were published in McQuade, K., Calzetti, D., & Kinney, A.L. (1995), *Astrophysical Journal Supplement* (ApJS), vol. 97, page 331 (though these authors did not try to determine Hubble's Constant from their spectra). The files are in ASCII format. The name of each file is the name of the corresponding galaxy + the extension ".txt". The ASCII files contain two columns: The first is the wavelength in Ångstrom, the second is the flux in units of erg/cm^2/s/Å.

The total wavelength range covered by the spectra is typically 1230-7500 Å. The UV spectra are from the Atlas of archival IUE spectra by Kinney et al. (1993, ApJS, 86, 5). The optical spectra were observed with the Kitt-Peak 0.9-m telescope with the infrared spectrograph during September 1991 and July 1992. The spectral resolution is 5-6 Å for the IUE spectra and 10 Å for the Kitt Peak spectra.

Your galaxy spectrum reading procedure (rdgalspec) has been supplied to you and is already included in the driver procedure (*i.e.*, hubble).

Many times you will have scientific data that is expressed in very large and/or very small numbers. When plotting such numbers, for appearance, it is often best to multiply the data by a *scale factor* and show that scale factor in the label title for a given plot (as shown in both Figures 1 and 2 where the flux on the y-axis has been multiplied by 10^{13}, hence the flux is labeled as 10^{-13} erg/s/cm^2/Å). In the driver procedure hubble, I set the scale factor with the routine galflscale which I have supplied to you on the Course Computer Project web page.

4.4 Plotting the Spectra

The driver procedure hubble.pro will make plots on the terminal screen and send them to two *hardcopy* files: an *encapsulated* postscript file (which is to be used in your project paper) and a *normal* postscript file (which can be sent directly to a printer). Since you will have both absorption line and emission line spectra, the program will show you the whole spectrum first, then you interactively tell the code which type of spectrum you currently have. I have included a nice little GUI widget procedure that I wrote, spectype.pro, that will help you select the spectrum type.

Once you determine the spectrum type, the code will load the appropriate lines (emission or absorption), that is, their IDs and their wavelengths into the **spectrum analysis** portion of the code.

4.5 Spectrum Analyses

You will be fitting **Gaussian profiles** to either your emission lines (*i.e.*, H-β, [O III]λ4959, and [O III]λ5007) or absorption lines (*i.e.*, Ca II H & K). The hubble.pro code uses the IDL function GAUSSFIT to do this. To use GAUSSFIT, you should only pass the spectral data points that are in the line of interest and 5 to 10 points to the either side of the line. Items that are retrieved (or calculated) include the **FWHM** ($\Delta\lambda_{1/2}$) and the **line center** ($\lambda_\circ$) of the

Gaussian. Make sure you compare your calculated FWHM to the telescope/spectroscope's spectral resolution and comment on this comparison. You will also need to figure out the **uncertainty** in the position of line center. The code has been set up to do this for you.

4.6 Determining Hubble's Constant

Now that you have the redshifts (from the Doppler effect and the observed line center) and the distances, and their corresponding uncertainties, you can use the method of **least squares** to determine the slope of the line. The hubble.pro code uses the IDL function LINFIT. LINFIT will return the y-intercept and the slope of the line in a two-element array. One can also retrieve the χ^2, standard deviations, and uncertainties of both of these parameters from various keyword settings. The actual uncertainty will depend not only on these uncertainties, but on the uncertainties in the distance and redshifts. See §4.7 below for details.

Your code will produce a graph that shows the least squares fit to your data (including uncertainties in each data point) and return Hubble's Constant and its uncertainty.

4.7 Calculating the Uncertainty for Hubble's Constant

To calculate the total uncertainty of the Hubble Constant you essentially just use Eq. (V-48) in your notes with the partials determined in the same manner that they were determined in Eqs. (V-38) through (V-40). There are three sets of uncertainties that have to be combined to calculate the lone uncertainty in $H_\circ$.

a) Uncertainties in wavelength (*i.e.*, the exact center of the Gaussian fit). Let's say, for example, that each spectrum has 3 lines that we are measuring. The GAUSSFIT function in IDL will give us 3 different line centers for these lines in the A[1] parameter (*e.g.*, A1[1], A2[1], A3[1], that is, the median of the Gaussian is the predicted line center). Now let's say that these lines have rest wavelengths of W1, W2, and W3. The Doppler effect gives (A[1] – W) / W = V / C, where V is the radial velocity and C is the speed of light (make sure you use a fairly accurate value for C = 2.997925×10^5 km/s). Now as you can see, you will get 3 different velocities from your 3 different DELTAWAVE = A[1] – W (= delta lambda) with Vi = (DELTAWAVE / Wi) $*$ C, "i" indicates each line in a given spectrum. Your velocity for that galaxy will be Vave = (V1 + V2 + V3) / 3, and your uncertainty in velocity for that galaxy is VSIGMAj = MAX(| Vi – Vave |). Now each galaxy will have its own uncertainty for velocity. At this point, calculate the total "normalized" uncertainty for velocity SVNORM = SQRT(SUM [(VSIGMAj / Vave-j)^2]), where "j" represents each galaxy and SUM simply means add all of these terms together (one could make use of the TOTAL function in IDL for this) and $\wedge$ is the IDL math symbol for *raise to this power*.

b) Uncertainty in distance (given in the galdist.txt file). Once again, we want a total normalized uncertainty here, so SDNORM = SQRT(SUM [(DISTj / UNC_DISTj)^2]). Note that we are using the square root of the sum of the squares because the data is uncorrelated.

c) Uncertainty in the slope of the fitted line. The method of least squares is handled with the IDL function called LINFIT. An example call to LINFIT would be

```
MFIT = LINFIT(DIST, Vave, SIGMA=SLEASTSQ).
```

LINFIT will return MFIT which is a two-element array containing the y-intercept (MFIT[0]) and the slope (i.e., Hubble's Constant, MFIT[1]) of the fitted straight line. In this command, DIST is an input array containing all of our DISTj data for all "j" galaxies and Vave is an input array containing all of the average velocity measurements (which has to be a one-to-one correlation with the data in DIST). The SLEASTSQ variable passed through the keyword SIGMA will be a two-element array upon output containing the uncertainties in the y-intercept (SLEASTSQ[0]) and the slope (*i.e.*, Hubble's Constant uncertainty from the fit, SLEASTSQ[1]). After this function, one would need to normalize this uncertainty with SHNORM = SLEASTSQ[1] / MFIT[1].

d) The Total Uncertainty in $H_\circ$. The total normalized uncertainty is then SNORM = SQRT(SVNORM^2 + SDNORM^2 + SHNORM^2) and the final uncertainty in your calculated Hubble's Constant is given by DELTAH = SNORM $*$ MFIT[1].

Since each parameter in the uncertainty calculation has different units, one must use the normalization technique described above. Then multiply this normalized uncertainty by the actual calculated Hubble Constant to convert this normalized uncertainty to the proper units.

5 Useful IDL Commands that Are Used in the Codes

Here is a list of some of the IDL procedures that are used in the codes posted on the Course Computer web site. Use the help facility in IDL for more information on these functions, commands, and internal procedures.

Input, Output, and **Plotting**: SET_PLOT, DEVICE, PLOT, OPLOT, PLOTERR, OPLOTERR, XYOUTS, POLYFILL, READ, PRINT, READF, and PRINTF.

Generic: N_ELEMENTS, WHERE, FLTARR, INTARR, FINDGEN, INDGEN, and FINDFILE.

String Manipulation: STRLEN, STRMID, STRUPCASE, STRLOWCASE, and STRPUT.

Math: MAX, MIN, LOG, ALOG10, EXP, SIN, COS, TAN, LINFIT, GAUSSFIT, CURVEFIT, COMFIT, and REGRESS.

6 Honors and Graduate Student Additional Work

Besides using the Hubble's Law plot that hubble.pro generates in your paper, graduate students, and those students taking the honors section of this course (*i.e.*, PHYS-4007-088) need to include two additional plots in your paper. Include an encapsulated postscript plot of the fitting of a Gaussian profile to one emission spectral line of one emission-line galaxy spectrum that you downloaded and a second plot of a Gaussian fit to one absorption line from one of the absorption-line galaxy spectra. As you will see from running the code, this "fitting" is demonstrated by using a solid line for the spectrum line and asterisk markers (using the PSYM=2 keywords in your OPLOT command) for the Gaussian fit. This "hardcopy" plot can be generated by making an additional pass through this portion code and sending the plot to an encapsulated postscript file instead of the terminal. This only needs to be done once for one emission line and once for one absorption line. Make sure you reset the graphics output to the terminal (using SET_PLOT) once the hardcopy file is made before continuing on with the code. Study how this is done when making the final v_r as a function of d plot for Hubble's Law near the end of the code.

7 Writing the Final Manuscript

The Final Manuscript must be at least 5 pages in length (not including references, tables, figures, and code listing) for undergraduate students and 10 pages in length for honors and graduate students. Your references, figures, and tables should take up no more than an additional 5 pages. **Note that your manuscript must be written in LaTeX!** You will need to follow the same style used in profession scientific journals — break the paper up into sections:

- **Abstract**: Summary of this work and the results obtained.

- **Introduction**: Contains a discussion of what Hubble's Law is, why it is importance to science, and what Hubble's Constant tells us about the Universe.

- **Analysis**: Contains a discussion on what steps you took to analyze data, including steps you took to ascertain the uncertainty in Hubble's Constant (see above). If you decide to do the Extra Credit described below, that figure should be included in this section.

- **Results and Conclusion**: Present your Hubble's Law plot and mention what values you got for $H_\circ$ and its uncertainty. Give a thorough description of what this plot tells you. You might also want to mention what age your value of $H_\circ$ gives for the age of the Universe. Include some concluding remarks about this project and what you learned from it.

Feel free to use the professional journal style LaTeX template file (*i.e.*, template.tex) for your manuscript and this Instruction Packet (file stuhubble.tex) as a guide to help you write your manuscript. Email the LaTeX file of your manuscript, the PDF file generated from it, and your modified hubble.pro code to me by the project due date.