PHYS-4617: Quantum Physics
Problem Set 2 (Due: 17 March 2005)

1. (30 pts) Starting with the expectation value of momentum (see Eq. II-47 in your notes),

$$\langle p \rangle = -i\hbar \int_{-\infty}^{+\infty} \left(\Psi^* \frac{\partial \Psi}{\partial x} \right) dx,$$

prove Ehrenfest's theorem,

$$\frac{d\langle p \rangle}{dt} = \langle -\frac{\partial V}{\partial x} \rangle,$$

using integral calculus with quantum mechanics.

2. (25 pts) Calculate $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, $\langle p^2 \rangle$, σ_x, and σ_p, for the nth state of the infinite square well. Check that the uncertainty principle is satisfied. Which state come closest to the uncertainty limit?

3. (25 pts) A particle in the harmonic oscillator potential has the initial wave function

$$\Psi(x, 0) = A[\psi_0(x) + \psi_1(x)]$$

for some constant A.

(a) Normalize $\Psi(x, 0)$.

(b) Find $\Psi(x, t)$, and $|\Psi(x, t)|^2$.

(c) Find the expectation value of x as a function of time. Notice that it oscillates sinusoidally. What is the amplitude of the oscillation? What is the (angular) frequency?

(d) Use your result in (c) to deteremine $\langle p \rangle$.