CSCI 1900
Discrete Structures

Probability
Reading: Kolman, Section 3.4

Probability Theory

There are two types of experiments:
– Deterministic – the outcome is always the same
– Probabilistic – the outcome could be any of a number of possible outcomes

Now that we know how to "count" using the multiplication principle, permutations, and combinations, we can figure out the probability of a certain outcome for probabilistic experiments.
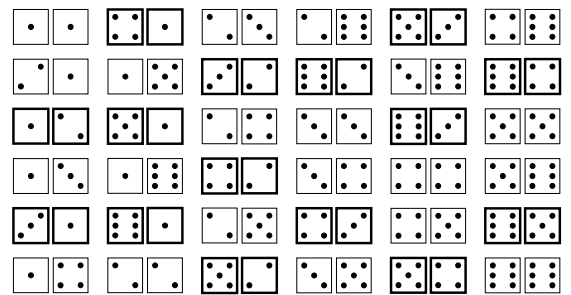

Sample Spaces

• The set of all possible outcomes of a probabilistic experiment is called the **sample space**.
• Tossing a pair of dice results in one of $_7C_2$.
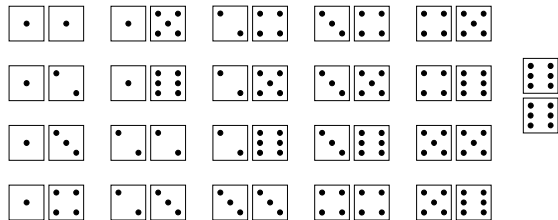
Sample Spaces (continued)

The previous slide doesn't take into account the fact that in all but 6 cases, each pattern could be the result of 2 different rolls

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Sample Spaces (continued)

• When determining the size of the sample space, you need to be sure of the method by which the sample space is created.
• Rolling dice – duplicates allowed, order matters (multiplication principle)
• Poker – duplicates not allowed, order doesn't matter (combinations)

Events

• An event is a set of outcomes that satisfy a statement (remember that a statement is something that must be true or false).
• Poker example – a statement about a hand of poker might be that the hand contained four of a kind.
• Dice example – a statement about a roll of the dice might be that a pair came up or that the sum of the dots equals 7.

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Events (continued)

- The list of all possible outcomes that satisfies an event makes a set.
- The events for which a roll of dice results in a pair is {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}.
- The events for which a roll of dice results in a sum of 7 is {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}.

Events (continued)

- Since an event is a set, then all of the operations on sets can apply to events.
- The events for which a roll of a pair of dice is either a pair or the sum equals 7 is {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}.
- The events for which a roll of a pair of dice is a pair and the sum equals 7 is the empty set.

Equally Likely Outcomes

- Assuming that any outcome is equally likely, i.e., there is no bias towards a particular subset of outcomes, then the probability of any outcome from a sample space with n possible outcomes is:

$$1/n$$

- The probability of an outcome from the event set, E, containing $|E|$ possible outcomes is:

$$|E|/n$$

Poker Odds Calculation

- Total possible hands = $_{52}C_5$ = 2,598,960
- Royal Straight Flush → 4 possible hands
  Odds are 4 in 2,598,960 → 1:649,740
- Straight Flush → 40 possible hands
  Odds are 40 in 2,598,960 → 1:64,974
- Four Aces → 48 possible hands
  Odds are 48 in 2,598,960 → 1:54,145
- Four of a kind → $_{13}C_1 \times _{48}C_1$ = 624 hands
  Odds are 624 in 2,598,960 → 1:4,165
- Full house → $_{13}C_1 \times _4C_3 \times _{12}C_1 \times _4C_2$ = 3,744
  Odds are 3,744 in 2,598,960 → 1:694

In-Class Exercise

- Is it worth it to play PowerBALL?