

CSCI 1900 Discrete Structures

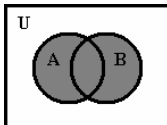
Operations on Sets
Reading: Kolman, Section 1.2

Operation on Sets

- An operation on a set is where two sets are combined to produce a third

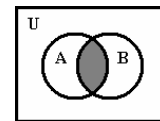
Union

- $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- Example:
Let $A = \{a, b, c, e, f\}$ and $B = \{b, d, r, s\}$
 $A \cup B = \{a, b, c, d, e, f, r, s\}$
- Venn diagram



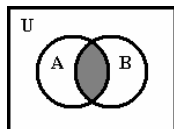
Intersection

- $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- Example:
Let $A = \{a, b, c, e, f\}$,
 $B = \{b, e, f, r, s\}$, and $C = \{a, t, u, v\}$.
 $A \cap B = \{b, e, f\}$
 $A \cap C = \{a\}$
 $B \cap C = \{\}$
- Venn diagram

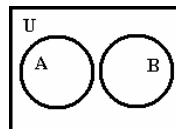


Disjoint Sets

Disjoint sets are sets where the intersection results in the empty set



Not disjoint



Disjoint

Unions and Intersections Across Multiple Sets

Both intersection and union can be performed on multiple sets

- $A \cup B \cup C = \{x \mid x \in A \text{ or } x \in B \text{ or } x \in C\}$
- $A \cap B \cap C = \{x \mid x \in A \text{ and } x \in B \text{ and } x \in C\}$
- Example:
 $A = \{1, 2, 3, 4, 5, 7\}$, $B = \{1, 3, 8, 9\}$, and $C = \{1, 3, 6, 8\}$.
 $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
 $A \cap B \cap C = \{1, 3\}$

Complement

- The complement of A (with respect to the universal set U) – all elements of the universal set U that are **not** a member of A .
- Denoted \bar{A}
- Example: If $A = \{x \mid x \text{ is an integer and } x \leq 4\}$ and $U = \mathbb{Z}$, then $\bar{A} = \{x \mid x \text{ is an integer and } x > 4\}$
- Venn diagram

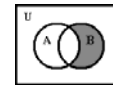


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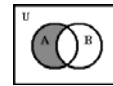
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Complement “With Respect to…”

- The complement of B with respect to A – all elements belonging to A , but not to B .
- It's as if U in the complement is replaced with A .
- Denoted $A - B = \{x \mid x \in A \text{ and } x \notin B\}$
- Example: Assume $A = \{a, b, c\}$ and $B = \{b, c, d, e\}$
 $A - B = \{a\}$
 $B - A = \{d, e\}$
- Venn diagram



$B - A$



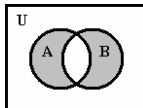
$A - B$

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Symmetric difference

- Symmetric difference – If A and B are two sets, the symmetric difference is the set of elements belonging to A or B , but not both A and B .
- Denoted $A \oplus B = \{x \mid (x \in A \text{ and } x \notin B) \text{ or } (x \in B \text{ and } x \notin A)\}$
- $A \oplus B = (A - B) \cup (B - A)$
- Venn diagram



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Algebraic Properties of Set Operations

- Commutative properties
 $A \cup B = B \cup A$
 $A \cap B = B \cap A$
- Associative properties
 $A \cup (B \cap C) = (A \cup B) \cap C$
 $A \cap (B \cup C) = (A \cap B) \cup C$
- Distributive properties
 $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

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More Algebraic Properties of Set Operations

- Idempotent properties
 $A \cup A = A$
 $A \cap A = A$
- Properties of the complement
 $\overline{\overline{A}} = A$
 $A \cup \bar{A} = U$
 $A \cap \bar{A} = \emptyset$
 $\bar{\emptyset} = U$
 $\bar{U} = \emptyset$
 $\overline{A \cup B} = \bar{A} \cap \bar{B}$ -- De Morgan's law
 $\overline{A \cap B} = \bar{A} \cup \bar{B}$ -- De Morgan's law

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More Algebraic Properties of Set Operations

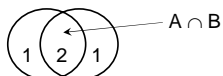
- Properties of a Universal Set
 $A \cup U = U$
 $A \cap U = A$
- Properties of the Empty Set
 $A \cup \emptyset = A$ or $A \cup \{\} = A$
 $A \cap \emptyset = \emptyset$ or $A \cap \{\} = \{\}$

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The Addition Principle

- The Addition Principle associates the cardinality of sets with the cardinality of their union
- If A and B are finite sets, then
 $|A \cup B| = |A| + |B| - |A \cap B|$
- Let's use a Venn diagram to prove this:



- The Roman Numerals indicate how many times each segment is included for the expression $|A| + |B|$
- Therefore, we need to remove one $|A \cap B|$ since it is counted twice.

Addition Principle Example

- Let $A = \{a, b, c, d, e\}$ and $B = \{c, e, f, h, k, m\}$
- $|A| = 5$, $|B| = 6$, and $|A \cap B| = |\{c, e\}| = 2$
- $|A \cup B| = |\{a, b, c, d, e, f, h, k, m\}|$
 $|A \cup B| = 9 = 5 + 6 - 2$
- If $A \cap B = \emptyset$, i.e., A and B are disjoint sets, then the $|A \cap B|$ term drops out leaving $|A| + |B|$